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Static bending analysis of two-directional functionally graded beam using simple Timoshenko beam elements

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ABSTRACT

This article presents the static bending of two-directional functionally graded (FG) beam by using simple Timoshenko beam elements. The Matlab code developed based on the finite element formulation is validated by solving two-directional FG beam problems under distributed load and two boundary conditions. Numerical results which are in terms of maximum normalized transverse deflections are compared with the analytical solutions and the results from previous studies. Besides, the shapes of transverse deflection and rotation along the length of beams are also depicted in this article to provide specific views about the static behavior of proposed structure.

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1. INTRODUCTION

In recent years, the use of the structures which are made of functionally graded materials (FGM) have been increasing in many modern engineering applications due to varying material properties over dimensions which allow to enhance the strength of material, high resistance to temperature shocks and high strength to weigth ratio. Authors devoted a considerable number of studies to predict and to understand the mechanics of the functionally graded structures during the last decade [1-10]. In [1], the main idea of this paper's developed method was the control over the produced gradient and did not require burning binder phase. The element free Galerkin method and Galerkin formulation for functionally graded beams were used to free vibration analysis as in [2]. Besides, overviews of FGM beam dynamics in thermal environment are presented by Parida *et al.* [3]. The simply supported beam is only considered and carried out by compelling the finite element procedure. The effects of porosity on bending static analysis of functionally graded (FG) beams was firstly introductioned by using a refined mixed finite element beam model. Two different types of porosity namely even and uneven distributions were also considered in [4]. Due to the task related to static analysis with probabilistic parameters for Euler-Bernoulli type functionally graded structures, Wu *et al.* investigated clearly above issue [5], and so on. In addition, the finite element method is an effective method for structural analysis like beams, plates and shells [11-25]. Among many different beam theories, the simple Timoshenko beam model helps us to reduce the computational cost with the resulting error within the allowable range. This article will investigate the bending behavior of two-directional functionally graded beams by using finite element procedure with simple Timoshenko beam elements respectively.

It is divided four sections. Section 1 gives the introduction as above. Section 2 presents the formulations as well as Section 3 shows some essential results. Finally, a few comments are also given in Section 4.

2. TWO-DIRECTIONAL FG BEAM AND FINITE ELEMENT PROCEDURE

A two-directional FG beam of length *L*, width *b* and thickness *h* as shown in Figure 1 is studied in this article. It is made by continuously changing from ceramic to metal phases through x and z directions.

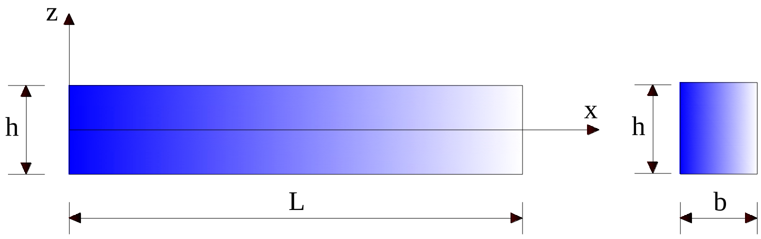

Figure 1. A two-directional FG beam

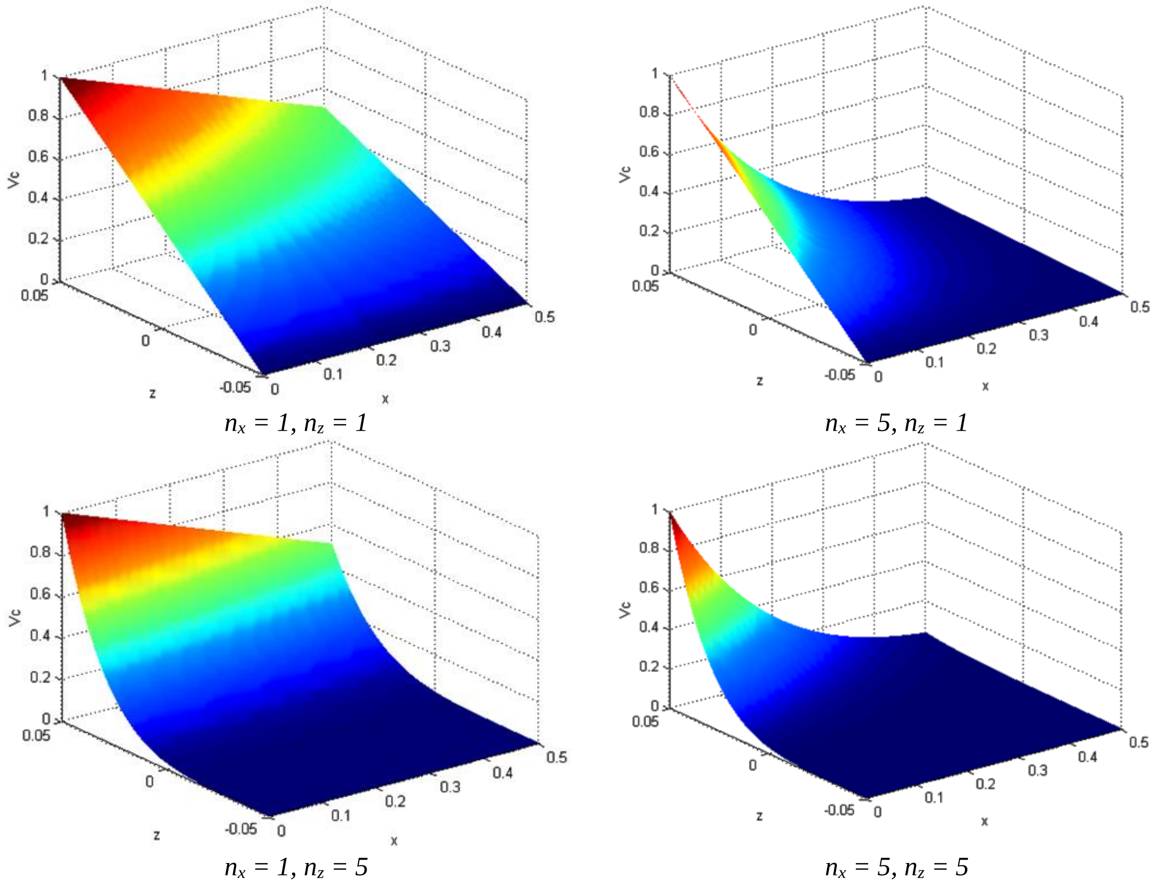

Figure 2. The modification of volume fraction of the ceramic

According to the power law form, the volume fraction of the ceramic can be presented in Eq. (1). The modification of this parameter is also depicted in Figure 2 respectively.

$$V_c(x,z)=\left(1-\frac{x}{2L}\right)^{n_x}\left(\frac{1}{2}+\frac{z}{h}\right)^{n_z} \tag{1}$$

The material properties can be calculated as belows

$$E(x,z)=(E_c-E_m)\left(1-\frac{x}{2L}\right)^{n_x}\left(\frac{1}{2}+\frac{z}{h}\right)^{n_z}+E_m \tag{2}$$

$$G(x,z)=(G_c-G_m)\left(1-\frac{x}{2L}\right)^{n_x}\left(\frac{1}{2}+\frac{z}{h}\right)^{n_z}+G_m \tag{3}$$

and

$$\nu(x,z)=(\nu_c-\nu_m)\left(1-\frac{x}{2L}\right)^{n_x}\left(\frac{1}{2}+\frac{z}{h}\right)^{n_z}+\nu_m \tag{4}$$

Based on finite element method (FEM), the degrees of freedom associated with a node of a simple Timoshenko beam element are a transverse displacement and a rotation as depicted in Figure 3. Using the principles of simple beam theory, the beam element stiffness matrix will be derived

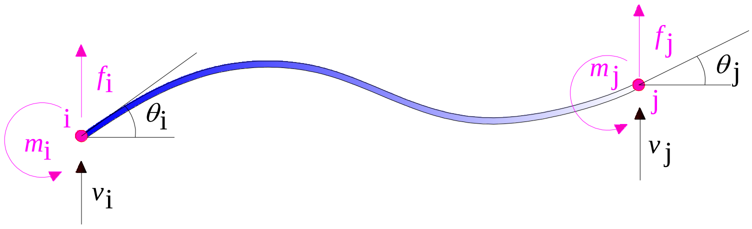

Figure 3. A simple Timoshenko beam

$$K_e = \frac{E_e I_e}{L_e^3(1+\Phi)}\begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & (4+\Phi)L_e^2 & -6L_e & (2-\Phi)L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & (2-\Phi)L_e^2 & -6L_e & (4+\Phi)L_e^2 \end{bmatrix} \quad (5)$$

with

$$\Phi = \frac{12E_e I_e}{G_e k A_e L_e^2} \quad (6)$$

and k =5/6 is called the shear correct factor.

According to the principle of minimum total potential energy, the element equation can be described as

$$\frac{E_e I_e}{L_e^3(1+\Phi)}\begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & (4+\Phi)L_e^2 & -6L_e & (2-\Phi)L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & (2-\Phi)L_e^2 & -6L_e & (4+\Phi)L_e^2 \end{bmatrix}\begin{Bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{Bmatrix} = \begin{Bmatrix} f_i \\ m_i \\ f_j \\ m_j \end{Bmatrix} \quad (7)$$

After assembly, the bending parameters can be obtained by solving the following equation

$$\mathbf{Kd} = \mathbf{F} \quad (8)$$

By using two letters ‘C’ and ‘S’ refer to the clamped and simply supported, all boundary conditions can be revealed as belows

$$v(0) = \theta(0) = 0,\ v(L) = \theta(L) = 0 \quad (9)$$

$$v(0) = \theta(0) = 0,\ v(L) = 0 \quad (10)$$

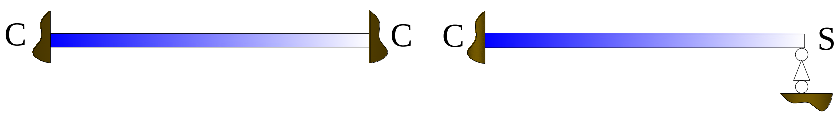

Figure 4. Two boundary conditions (CC) and (CS)

The finite element procedure can be reached as below

- Input data: material properties and geometric data.
- Calculating constitutive matrix.
- Loop over elements: calculating element force vector and element stiffness matrix.
- Assembling all parts in the global coordinate system
- Applying two boundary conditions (CC) and (CS).
- Solving equation for static bending
- Display transverse displacements and rotations at nodes of structure.

3. RESULTS

In this section, some numerical tests are given to verify the applicability of proposed method. A two-directional FG beam with length L = 1m, b = 0.1m, length to thickness ratio L/h = 5 and distributed load $q = 10^4$ N/m is considered and the material properties can be seen in Table 1 for two-phase.

Table 1. The material properties

Al_2O_3	E_c = 380GPa	$\nu_c = 0.3$
Al	E_m = 70GPa	$\nu_m = 0.3$

Table 2. The comparison of normalized tranverse deflection of (CS) two-directional FG beam

L/h = 5	n_x					
n_z	0		1		5	
	SSPH	Present	SSPH	Present	SSPH	Present
0	1.1972	1.1709	1.4733	1.4125	2.7279	2.4549
1	2.0219	1.9541	2.3975	2.2820	3.7281	3.4284
5	2.8689	2.7577	3.2817	3.1190	4.4918	4.1899

Table 3. The comparison of normalized tranverse deflection of (CC) two-directional FG beam

L/h = 5	n_x					
n_z	0		1		5	
	SSPH	Present	SSPH	Present	SSPH	Present
0	0.5755	0.6010	0.7352	0.7663	1.4013	1.4269
1	0.9722	0.9953	1.1871	1.2159	1.8722	1.9118
5	1.3795	1.3998	1.6139	1.6402	2.2239	2.2737

Firstly, the normalized transverse deflection at *L*/2 is formulated by $\bar{v}(L/2) = 100E_m bh^3 v(L/2)/q/L^4$ as well as all results related to this proposed method for two boundary conditions (CC) & (CS) are compared with other results from [17] by author Karamanli as shown in Tables 2 & 3. In this reference, the Symmetric Smoothed Particle Hydrodynamics (SSPH) method was used to calculate bending deflection respectively. It can be seen that the results obtained from this paper are completely approximate with the results of SSPH method. The relative error among these results can be explained by using different approaches.

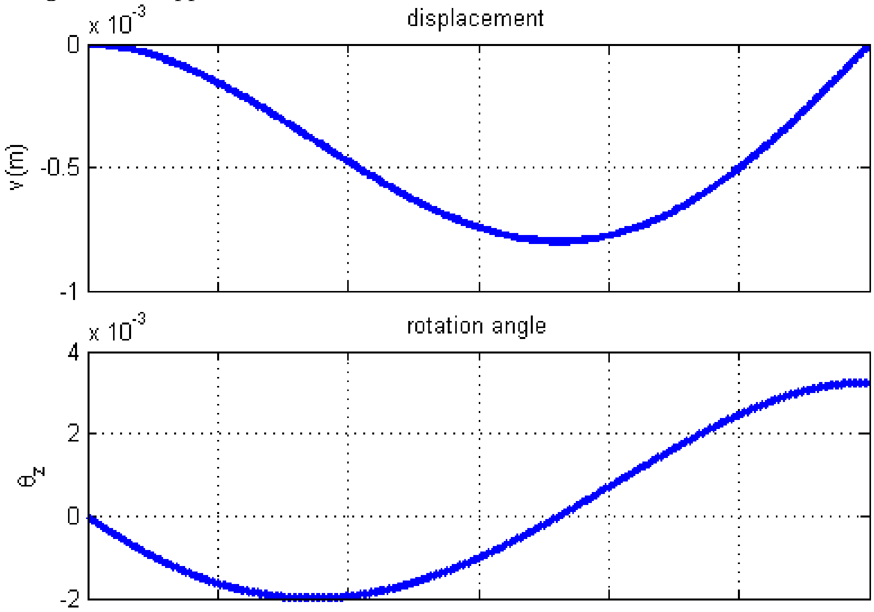

Figure 5. The displacement along the length and rotation angle of (CS) two-directional FG beam with $n_x = n_y = 5$

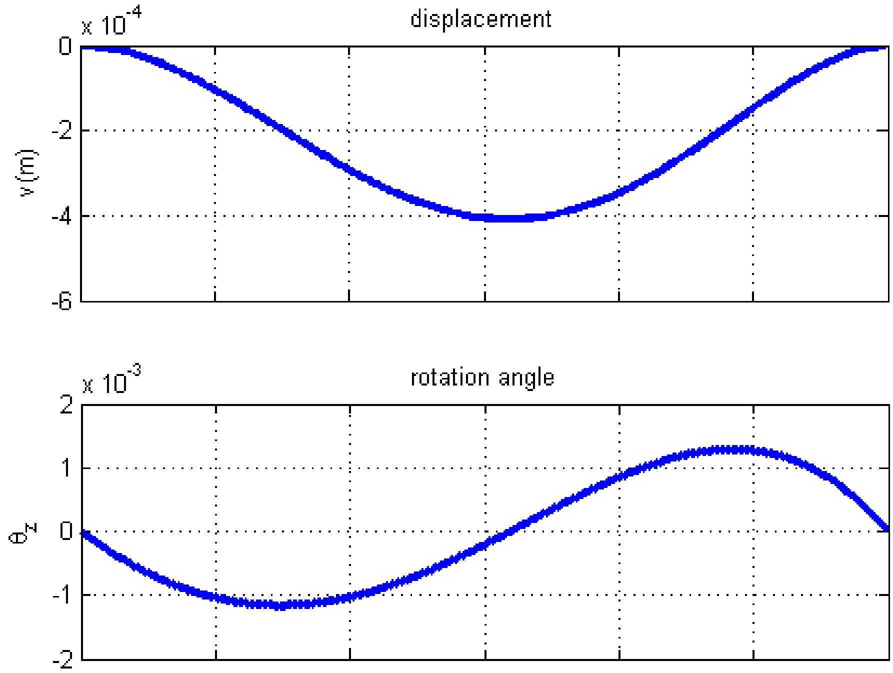

Figure 6. The displacement along the length and rotation angle of (CC) two-directional FG beam with $n_x = n_y = 5$

Figures 5 & 6 depict the displacement and rotation angle along the length of (CC) & (CS) two-directional FG beams with two factors $n_x = n_z = 5$.

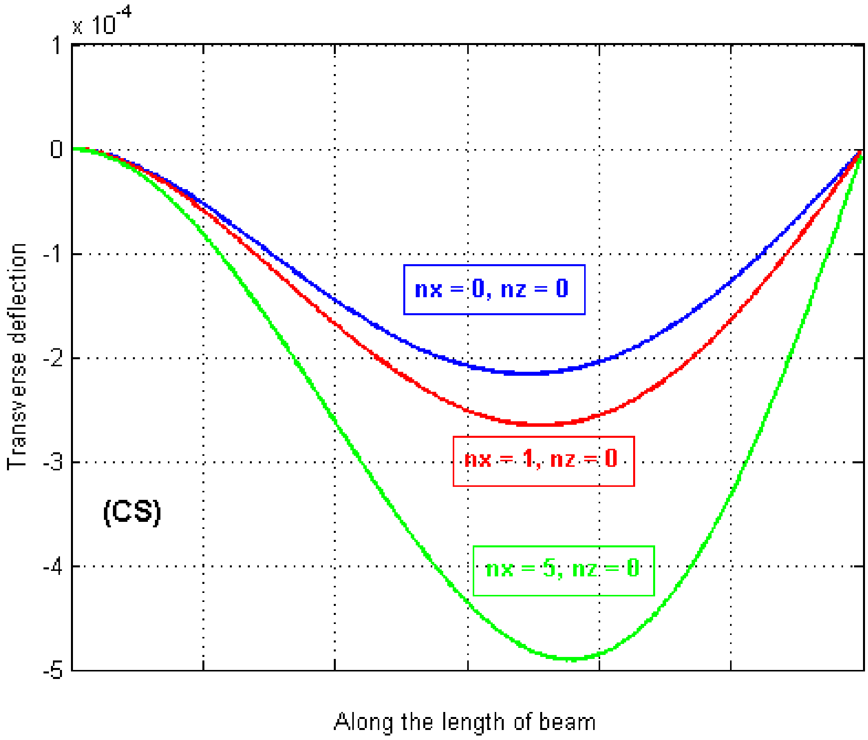

Figure 7. The transverse deflection along the length of (CS) two-directional FG beam by changing n_x

Finally, to investigate the influence of n_x, n_y on the deflection of two-directional FG beams for two boundary conditions (CC) & (CS), Figures 7-10 present curves showing transverse deflections along the length of beams. It is clear that the deflection value increases when n_x or n_z increases.

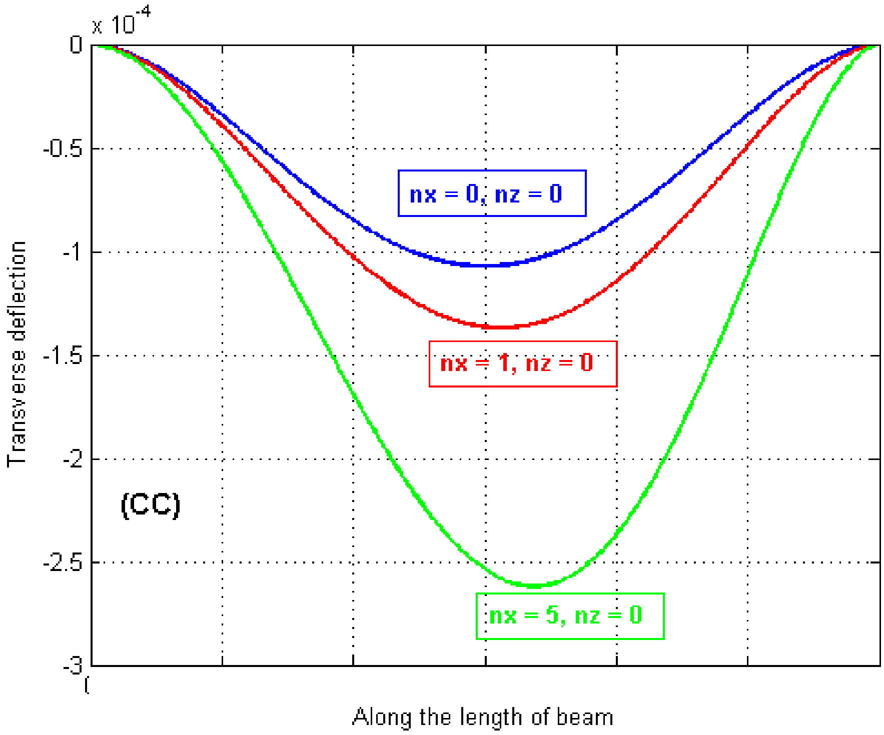

Figure 8. The transverse deflection along the length of (CC) two-directional FG beam by changing n_x

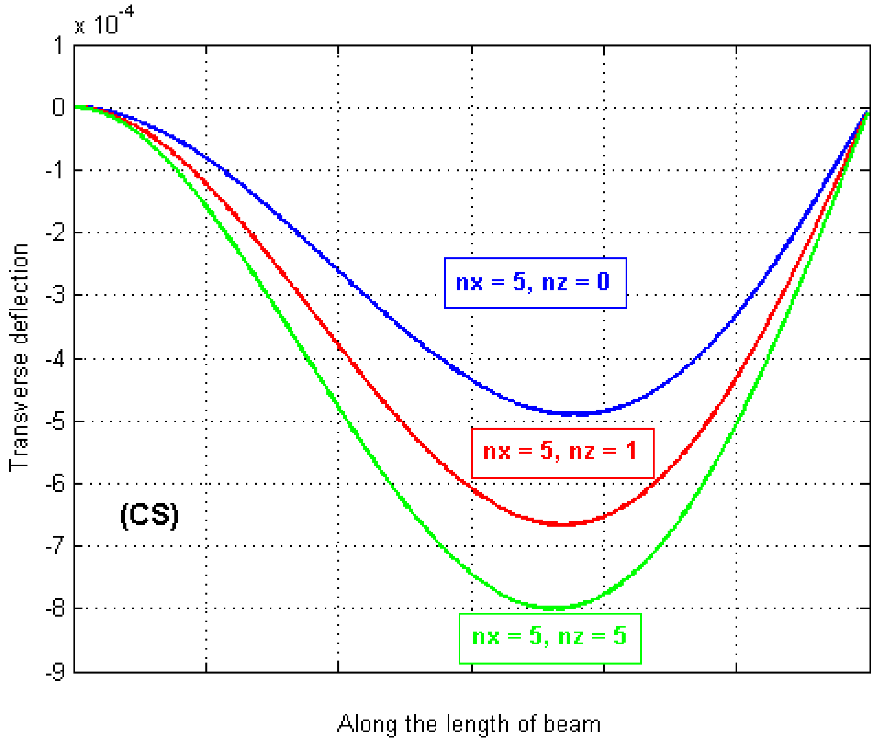

Figure 9. The transverse deflection along the length of (CS) two-directional FG beam by changing n_z

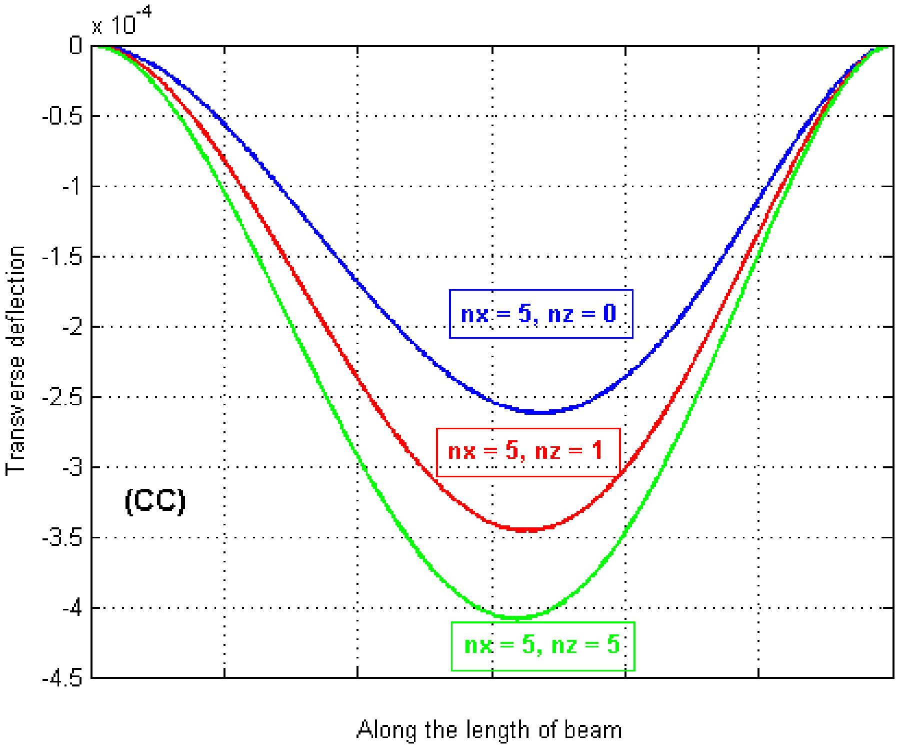

Figure 10. The transverse deflection along the length of (CC) two-directional FG beam by changing n_z

4. CONCLUSION

In this article, the bending behaviors of two-directional FG beams under two different types of boundary condition (CC), (CS) are studied. The simple Timoshenko beam elements based on the traditional finite element procedure are firstly used to analyze the bending behavior of two-directional FG beams and then the author proves the applicability of them through several numerical examples. The results of this article are in good agreement with other solutions in references.

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