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Measurement the effects of temperature and fiber orientation on vibration of functionally graded beam

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ABSTRACT

This paper concerned with analytical approach to study the thermal vibration of fiber orientation functionally graded (FOFG) beam, that fibers`oriented angles are variable and graded in the thickness direction of the beam. Uniform thermal distribution considered in the entire beam and properties of fiber orientation functionally graded (FOFG) beam considered as the temperature-dependent element. Symmetrical, asymmetrical, and classical distribution types for the mode of fiber angle presented in the thickness direction of the beam continuously. Equilibrium Equations derived from first- order shear deformation theory and Hamilton principle. Simply supported boundary condition is considered for both edges of the beam.Eneralized differential quadrature method usedto solve the system of coupled differential Equations. To study accuracy of the present analysis, a compression carried out with a known data. The results shows that different parameters such as thickness to radius ratio, effect of temperature variations, model of fibers angle variations and power-law index affected on the natural frequencies.

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1. INTRODUCTION

Composite or FGM beams characterized by continuous or discontinuous, uniaxially or spatially variable material properties [1]. Tanigawa [2] has compiled comprehensive lists of papers on the analytical models of the thermoplasticbehavior of FGMs.Chenaet. al [3] studied the energy flow analysis (EFA) method developed to predict the high frequency response of beams in a thermal environment. Vibration and thermal buckling behavior of sandwich beams with composite facings and viscoelastic core presented [4]. Hui-Shen Shen[5]studiedthelarge amplitude vibration, nonlinear bending and thermal postbuckling of functionally graded material (FGM) beams resting on an elastic foundation in thermal environments.A new kind of higher order shear deformation theory for functionally graded materials that explicitly couples the micro structural and macro structural effects expanded by Aboudi et al.[6] , Benatta et al. [7] and Sallai et al. [8] solved static bending deformations of simply supported FGM hybrid beams subjected to uniformly distributed transverse

loads analytically by using a higher-order shear deformation theory and gave numerical results for the deflection, and the transverse normal and the transverse shear stresses.
Kadoli et al. [9] by using the finite element method and the third-order shear deformation theory (TSDT) an analysed static bending deformation of FGM beams with different boundary conditions (BCs) at the edges and a uniform transverse load applied on the top surface. Li [10] studied static bending deformations and transverse vibrations of FGM Timoshenko beams and introduced a function to uncouple governing Equations for the deflection and the angle of rotation of a cross-section initially perpendicular to the neutral surface. Employing the same method, Huang and Li [11,12] analysed bending, buckling and free vibrations of FGM circular columns with material properties continuously varying in the radial direction by FSDT.Simsek [13] studied free vibrations of FGM beams using different higher-order shear deformation theories and derived governing Equations by using Hamilton's principle. Ke et al. [14,15] as well as Yang and Chen [16] studied free vibrations, buckling and post-buckling of FGM TBs containing open cracks by assuming an exponential variation of material properties in the thickness direction. Sankar [17] used the linear elasticity theory to analytically analyse deformations of simply supported FGM beams with Young's modulus varying exponentially in the thickness direction and subjected to symmetrical sinusoidal transverse loads. Pradhan andMurmu [18] studied Thermomechanical vibration analysis of functionally graded beams and functionally graded sandwich beams by considering the functionally graded material beams to be resting on variable (i) Winkler foundation and (ii) two-parameter elastic foundation and varying the material properties of these beams in the thickness direction. The governing differential Equations for beam vibration using the modified differential quadrature method.
Mostapha Raki et al.[19]analysed the thermal buckling of thin rectangular FGM plate. Considering thematerial properties vary as a power form of the thickness coordinate variable z and using the variational method.
The free vibration analysis of initially stressed thick simply supported functionally graded curved panel resting on two-parameter elastic foundation, subjected in thermal environment studied using the three-dimensional elasticity formulation by Farid et al. [20].
In this paper, Effects of fiber orientation and thermal on the natural frequencies of a functionally graded fiber beam (FOFG) with distributions of fiber orientation investigated. Continuously subjected to the uniform temperature distribution through the beam.Properties of the reinforced fiber oriented beam ,which changes in the form of fiber orientation angle, are considered temperature dependent and independent .And, it`s effects on the results compared with each other. The temperature obtained from equilibrium Equations by presuming the thermal strains in the form of pre-stress terms, and the effect of these pre-stresses on the governing Equations of motionwhich are derived by the FSD theory are considered.For obtaining the pre-stresses and also, the Equation of motions are solved by DQM method.The thermal vibration analysis performed in two stages: Initially, the stresses induced by the temperature increase in beam obtained, then these stresses imported as pre-stress in Equations of motion, and it caused the changes in natural frequencies of FOFG beam, finally.According to the shortage of conducted studies and papers about the thermal vibrationson the beam geometry and with these special properties of the used material, the procedure of extraction and solving the Equations on this geometry, performed by a special method here.

2. RESEARCH METHOD

2.1. Equations Of Equlibrium

Displacement field explained based on Equation. (3) :

$$\begin{aligned} U(x,z,t) &= u(x,t) - z\,\phi(x,t) \\ V &= 0 \\ W(x,z,t) &= w(x,t) \end{aligned} \tag{3}$$

Stress- strain relation in plane stress for an orthotropic material based on Equation. (4), whichthe thermal strains applied in it are as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \varepsilon_{xz} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \\ \alpha_z \\ 2\alpha_{xz} \end{Bmatrix} \Delta T \right) \tag{4}$$

In this Equation, ΔT is the temperature variations of beam, $\alpha_i\,(i = x, y, z)$ thermal expansion factor and $\overline{Q_{ij}}$ coefficients ofreduced stiffness matrix.
Coefficients of reduced stiffness matrixrelated tothe fibers angle position in each point of orthotropic material as Eq. (5)

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \\
\bar{Q}_{12} &= (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta+\cos^4\theta) \\
\bar{Q}_{16} &= (Q_{11}-Q_{12}-2Q_{66})\sin\theta\cos^3\theta + (Q_{12}-Q_{22}+2Q_{66})\sin^3\theta\cos\theta \\
\bar{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \\
\bar{Q}_{26} &= (Q_{11}-Q_{12}-2Q_{66})\sin^3\theta\cos\theta + (Q_{12}-Q_{22}+2Q_{66})\sin\theta\cos^3\theta \\
\bar{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta+\cos^4\theta)
\end{aligned} \tag{5}$$

Where:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}},\quad Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}},\quad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}},\quad Q_{66} = G_{12},$$

3.

The thermal expansion coefficients along the various geometrical directions based on Equation. (6) depended on the position of fibers angle and thermal expansion coefficients along the main directions.

$$\begin{aligned}
\alpha_x &= \alpha_1\cos^2\theta + \alpha_2\sin^2\theta \\
\alpha_z &= \alpha_1\sin^2\theta + \alpha_2\cos^2\theta \\
\alpha_{xy} &= (\alpha_1-\alpha_2)\sin\theta\cos\theta
\end{aligned} \tag{6}$$

The strain relation in displacement is as Equation. (7)

$$\varepsilon_{0x} = \frac{\partial U}{\partial x},\qquad \gamma_{0xz} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \tag{7}$$

Stress elements in displacements obtained by substituting the Eq. (3) into (7), and then in Eq. (4). Finally, equilibrium Equations by using the Hamilton`s principle obtained as Equation. (8) in resultant forces and moments.

$$\begin{aligned}
\delta u &= \frac{dN_x}{\partial x} = 0 \\
\delta w &= \frac{dV_x}{\partial x} = 0 \\
\delta\phi &= -\frac{dM_x}{\partial x} + V_x = 0
\end{aligned} \tag{8}$$

Although, the various governing boundary conditions at both ends of FOFG beam such as simplyin resultants obtained as Equations (9) from Hamilton principle.

Simply

$$w = 0,\quad N_x = 0,\quad M_x = 0 \tag{9}$$

boundary condition(S):

By substituting resultants in displacements *u, φ, w*, which are unknown, inEquations. (8-9), equilibriumEquations and boundary conditions will obtained.

The DQM numerical method used to solve the couple differential Equation system in displacement variants. By applying this method to equilibrium Equations and boundary conditions, it reached to Equations.10-11.

$$\begin{aligned}
&A_{11}\sum_{j=1}^{N}c_{ij}^{(2)}u_j - B_{11}\sum_{j=1}^{N}c_{ij}^{(2)}\phi_j + A_{44}\sum_{j=1}^{N}c_{ij}^{(2)}w_j - A_{44}\sum_{j=1}^{N}c_{ij}^{(1)}\phi_j = 0\\
&A_{44}\sum_{j=1}^{N}c_{ij}^{(2)}u_j - B_{44}\sum_{j=1}^{N}c_{ij}^{(2)}\phi_j + A_{55}\sum_{j=1}^{N}c_{ij}^{(2)}w_j - A_{55}\sum_{j=1}^{N}c_{ij}^{(1)}\phi_j = 0\\
&-B_{11}\sum_{j=1}^{N}c_{ij}^{(2)}u_j + D_{11}\sum_{j=1}^{N}c_{ij}^{(2)}\phi_j - B_{44}\sum_{j=1}^{N}c_{ij}^{(2)}w_j + A_{44}\sum_{j=1}^{N}c_{ij}^{(1)}u_j + A_{55}\sum_{j=1}^{N}c_{ij}^{(1)}w_j - A_{55}\phi_i\\
&= (A_{44}\alpha_x + A_{33}\alpha_z + 2A_{55}\alpha_{xz})\Delta T
\end{aligned} \tag{10}$$

4. Simply boundary condition (S-BC):

$$\begin{aligned}
&w_i = 0\\
&A_{11}\sum_{j=1}^{N}c_{ij}^{(1)}u_j - B_{11}\sum_{j=1}^{N}c_{ij}^{(1)}\phi_j + A_{44}\sum_{j=1}^{N}c_{ij}^{(1)}w_j - A_{44}\phi_i - (A_{11}\alpha_x(z) + A_{22}\alpha_z(z) + 2A_{44}\alpha_{xz}(z))\Delta T = 0\\
&B_{11}\sum_{j=1}^{N}c_{ij}^{(1)}u_j - D_{11}\sum_{j=1}^{N}c_{ij}^{(1)}\phi_j + B_{44}\sum_{j=1}^{N}c_{ij}^{(1)}w_j - B_{44}\phi_i - (B_{11}\alpha_x(z) + B_{22}\alpha_z(z) + 2B_{44}\alpha_{xz}(z))\Delta T = 0
\end{aligned} \tag{11}$$

5. In the presented Equations it defined the thermal terms appeared in the main Equations and boundary conditions. By solving this Equation and obtaining the variables of u,φ,w it can be possible to obtain the created stresses in the FOFG beam caused by variations of temperature based on Eq. (12) Which in these displacement Equations u^1,ϕ^1,w^1 are the obtained values from equilibrium, and it is clear the displacement is zero when the beam is in the environment temperature $\Delta T = 0$ and final the initial stress or the pre-stress will create and is equal to zero.These stresses will created and increased by the increase of the temperature.

$$\begin{aligned}
\sigma_{0x} &= \bar{Q}_{11}\left(\frac{\partial u^1}{\partial x} - z\frac{\partial \phi^1}{\partial x} - \alpha_x\Delta T\right) - \bar{Q}_{12}(\alpha_z\Delta T) - \bar{Q}_{16}\left(\phi^1 - \frac{\partial w^1}{\partial x} + 2\alpha_{xz}\Delta T\right)\\
\sigma_{0xz} &= \bar{Q}_{16}\left(\frac{\partial u^1}{\partial x} - z\frac{\partial \phi^1}{\partial x} - \alpha_x\Delta T\right) - \bar{Q}_{26}(\alpha_z\Delta T) - \bar{Q}_{66}\left(\phi^1 - \frac{\partial w^1}{\partial x} + 2\alpha_{xz}\Delta T\right)
\end{aligned} \tag{12}$$

2.2.Equation Of Motion

Now through the second stage, whichEquations of motion obtained by considering the created pre-stresses due to temperature, and investigating the effects of these pre-stresses on natural frequencies of beam.
Based on introducing Hamilton`s principle as Eq. (17), *K* and *U* are the kinematic and potential energies, respectively, which includes explained σ_{0x},σ_{0xz} pre-stresses in Eq. (16).

$$\begin{aligned}
&\int_{t_1}^{t_2}(\delta K - \delta U)dt\\
&\delta K = \int_V \rho\left(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t}\right)dV\\
&\delta U = \int_V [(\sigma_x + \sigma_{0x})\delta\varepsilon_x + (\sigma_{xz} + \sigma_{0xz})\delta\gamma_{xz}]dV
\end{aligned} \tag{13}$$

Despitethe existence of linear and nonlinearterms,introduced as Eq. (14).

$$\begin{aligned}
\varepsilon_x &= \frac{\partial U}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial x}\right)^2\right]\\
\gamma_{xz} &= \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} + \frac{\partial U}{\partial x}\frac{\partial U}{\partial z} + \frac{\partial V}{\partial x}\frac{\partial V}{\partial z} + \frac{\partial W}{\partial x}\frac{\partial W}{\partial z}
\end{aligned} \tag{14}$$

By substitution of Eq. (3) in Eq. (14), then in Eq. (4), and finally in Eq. (13), the governing Equations of motion and various boundary conditions on both two ends of FOFG beam with existence of pre-stresses. Forces and momentums as presented in Equations (19 to 22).

$$\delta u = 0 \Rightarrow \frac{\partial N_x}{\partial x} + N_{0x}\frac{\partial^2 u}{\partial x^2} - M_{0x}\frac{\partial^2 \phi}{\partial x^2} - V_{0x}\frac{\partial \phi}{\partial x} = I_0\frac{\partial^2 u}{\partial t^2} - I_1\frac{\partial^2 \phi}{\partial t^2}$$
$$\delta w = 0 \Rightarrow \frac{\partial V_x}{\partial x} + N_{0x}\frac{\partial^2 w}{\partial x^2} = I_0\frac{\partial^2 w}{\partial t^2} \quad (15)$$
$$\delta \phi = 0 \Rightarrow -\frac{\partial M_x}{\partial x} + V_x - M_{0x}\frac{\partial^2 u}{\partial x^2} + V_{0x}\frac{\partial u}{\partial x} + P_{0x}\frac{\partial^2 \phi}{\partial x^2} = I_2\frac{\partial^2 \phi}{\partial t^2} - I_1\frac{\partial^2 u}{\partial t^2}$$

For Simply Supported(S):

$$w = 0$$
$$-N_x - N_{0x}\frac{\partial u}{\partial x} + M_{0x}\frac{\partial \phi}{\partial x} + V_{0x}\phi = 0 \quad (16)$$
$$M_x + M_{0x}\frac{\partial u}{\partial x} - P_{0x}\frac{\partial \phi}{\partial x} - R_{0x}\phi = 0$$

Presented resultants in Eqs. (8 to 11), and (15-16) introduced as Eqs. (17)

$$\begin{Bmatrix} N_x \\ N_{0x} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_{0x} \end{Bmatrix} dz, \quad \begin{Bmatrix} M_x \\ M_{0x} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z \begin{Bmatrix} \sigma_x \\ \sigma_{0x} \end{Bmatrix} dz,$$
$$\begin{Bmatrix} V_x \\ V_{0x} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xz} \\ \sigma_{0xz} \end{Bmatrix} dz, \quad \begin{Bmatrix} P_{0x} \\ R_{0x} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} z^2\sigma_{0x} \\ z\sigma_{0xz} \end{Bmatrix} dz \quad (17)$$

As like the first stage, Equations of motion and the total boundary conditionsin Equations. (15-16) obtained in displacement fields as u, φ, w, and by substituting Equations. (18) instead of displacement parameters it will reach to a couple differential Equationin u, φ, w variables which solved by DQM method.

$$u(x,t) = u_0(x)e^{i\omega t}$$
$$w(x,t) = w_0(x)e^{i\omega t} \quad (18)$$
$$\phi(x,t) = \phi_0(x)e^{i\omega t}$$

Where ω are natural frequencies in above Equations.

2.3 DQM Discretized From Of The Governing Equations

The final Equations by applying DQM method obtained as Equations. (19 - 21)

$$A_{11}\sum_{j=1}^{N} c_{ij}^{(2)} u_{0j} + A_{11}\sum_{j=1}^{N} c_{ij}^{(2)} (\sum_{j=1}^{N} c_{ij}^{(1)} u_j^1)_i u_{0j} - B_{11}\sum_{j=1}^{N} c_{ij}^{(2)} (\sum_{j=1}^{N} c_{ij}^{(1)} \phi_j^1)_i u_{0j} + A_{44}\sum_{j=1}^{N} c_{ij}^{(2)} (\sum_{j=1}^{N} c_{ij}^{(1)} w_j^1)_i u_{0j} -$$
$$A_{44}\sum_{j=1}^{N} c_{ij}^{(2)} (\phi_i^1)_i u_{0j} - \sum_{j=1}^{N} c_{ij}^{(2)} (T_{44})_i u_{0j} - B_{11}\sum_{j=1}^{N} c_{ij}^{(2)} \phi_{0j} - A_{44}\sum_{j=1}^{N} c_{ij}^{(1)} \phi_{0j} - B_{11}\sum_{j=1}^{N} c_{ij}^{(2)} (\sum_{j=1}^{N} c_{ij}^{(1)} u_j^1)_i \phi_{0j} +$$
$$D_{11}\sum_{j=1}^{N} c_{ij}^{(2)} (\sum_{j=1}^{N} c_{ij}^{(1)} \phi_j^1)_i \phi_{0j} + \sum_{j=1}^{N} c_{ij}^{(2)} (T_{55})_i \phi_{0j} - B_{44}\sum_{j=1}^{N} c_{ij}^{(2)} (\sum_{j=1}^{N} c_{ij}^{(1)} w_j^1)_i \phi_{0j} + B_{44}\sum_{j=1}^{N} c_{ij}^{(2)} (\phi_i^1)_i \phi_{0j} - \quad (19)$$
$$A_{44}\sum_{j=1}^{N} c_{ij}^{(1)} (\sum_{j=1}^{N} c_{ij}^{(1)} u_j^1)_i \phi_{0j} + B_{44}\sum_{j=1}^{N} c_{ij}^{(1)} (\sum_{j=1}^{N} c_{ij}^{(1)} \phi_j^1)_i \phi_{0j} - A_{55}\sum_{j=1}^{N} c_{ij}^{(1)} (\sum_{j=1}^{N} c_{ij}^{(1)} w_j^1)_i \phi_{0j} + A_{55}\sum_{j=1}^{N} c_{ij}^{(1)} (\phi_i^1)_i \phi_{0j} -$$
$$\sum_{j=1}^{N} c_{ij}^{(1)} (T_{66})_i \phi_{0j} + A_{44}\sum_{j=1}^{N} c_{ij}^{(2)} w_{0j} = -I_{0i}\omega^2 u_{0i} + I_{1i}\omega^2 \phi_{0i}$$

$$A_{44}\sum_{j=1}^{N} c_{ij}^{(2)} u_{0j} - B_{44}\sum_{j=1}^{N} c_{ij}^{(2)} \phi_{0j} - A_{55}\sum_{j=1}^{N} c_{ij}^{(1)} \phi_{0j} + A_{55}\sum_{j=1}^{N} c_{ij}^{(2)} w_{0j} + A_{11}\sum_{j=1}^{N} c_{ij}^{(2)} (\sum_{j=1}^{N} c_{ij}^{(1)} u_j^1)_i w_{0j} - \quad (20)$$
$$B_{11}\sum_{j=1}^{N} c_{ij}^{(2)} (\sum_{j=1}^{N} c_{ij}^{(1)} \phi_j^1)_i w_{0j} + A_{44}\sum_{j=1}^{N} c_{ij}^{(2)} (\sum_{j=1}^{N} c_{ij}^{(1)} w_j^1)_i w_{0j} - A_{44}\sum_{j=1}^{N} c_{ij}^{(2)} (\phi_i^1)_i w_{0j} - \sum_{j=1}^{N} c_{ij}^{(2)} (T_{44})_i w_{0j} = -I_{0i}\omega^2 w_{0i}$$

$$
\begin{aligned}
&-B_{11}\sum_{j=1}^{N}c_{ij}^{(2)}u_{0j}+A_{44}\sum_{j=1}^{N}c_{ij}^{(1)}u_{0j}+A_{44}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}u_{j}^{1})_{i}u_{0j}-B_{44}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{j}^{1})_{i}u_{0j}+\\
&A_{55}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}w_{j}^{1})_{i}u_{0j}-A_{55}\sum_{j=1}^{N}c_{ij}^{(1)}(\phi_{i}^{1})_{i}u_{0j}-\sum_{j=1}^{N}c_{ij}^{(1)}(T_{66})_{i}u_{0j}-B_{11}\sum_{j=1}^{N}c_{ij}^{(2)}(\sum_{j=1}^{N}c_{ij}^{(1)}u_{j}^{1})_{i}u_{0j}+\\
&D_{11}\sum_{j=1}^{N}c_{ij}^{(2)}(\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{j}^{1})_{i}u_{0j}-B_{44}\sum_{j=1}^{N}c_{ij}^{(2)}(\sum_{j=1}^{N}c_{ij}^{(1)}w_{j}^{1})_{i}u_{0j}+B_{44}\sum_{j=1}^{N}c_{ij}^{(2)}(\phi_{i}^{1})_{i}u_{0j}+\sum_{j=1}^{N}c_{ij}^{(2)}(T_{55})_{i}u_{0j}-\\
&B_{44}\sum_{j=1}^{N}c_{ij}^{(2)}w_{0j}+A_{55}\sum_{j=1}^{N}c_{ij}^{(1)}w_{0j}+D_{11}\sum_{j=1}^{N}c_{ij}^{(2)}\phi_{0j}+D_{11}\sum_{j=1}^{N}c_{ij}^{(2)}(\sum_{j=1}^{N}c_{ij}^{(1)}u_{j}^{1})_{i}\phi_{0j}-H_{11}\sum_{j=1}^{N}c_{ij}^{(2)}(\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{j}^{1})_{i}\phi_{0j}-\\
&\sum_{j=1}^{N}c_{ij}^{(2)}(T_{77})_{i}u_{0j}+D_{44}\sum_{j=1}^{N}c_{ij}^{(2)}(\sum_{j=1}^{N}c_{ij}^{(1)}w_{j}^{1})_{i}\phi_{0j}-D_{44}\sum_{j=1}^{N}c_{ij}^{(2)}(\phi_{i}^{1})_{i}\phi_{0j}-A_{55}\phi_{0i}=-I_{2i}\omega^{2}\phi_{0i}+I_{1i}\omega^{2}u_{0i}
\end{aligned}
\tag{21}
$$

Also, boundary Equations obtained as Equations. (22)

Simply boundary condition:

$$
\begin{aligned}
&w_{0i}=0\\
&-A_{11}\sum_{j=1}^{N}c_{ij}^{(1)}u_{0j}-A_{11}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}u_{j}^{1})_{i}u_{0j}+B_{11}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{j}^{1})_{i}u_{0j}-A_{44}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}w_{j}^{1})_{i}u_{0j}+\\
&A_{44}\sum_{j=1}^{N}c_{ij}^{(1)}(\phi_{j}^{1})_{i}u_{0j}+\sum_{j=1}^{N}c_{ij}^{(1)}(T_{44})_{i}u_{0j}+B_{11}\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{0j}+B_{11}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}u_{j}^{1})_{i}\phi_{0j}-D_{11}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{j}^{1})_{i}\phi_{0j}+\\
&B_{44}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}w_{j}^{1})_{i}\phi_{0j}-B_{44}\sum_{j=1}^{N}c_{ij}^{(1)}(\phi_{i}^{1})_{i}\phi_{0j}-\sum_{j=1}^{N}c_{ij}^{(1)}(T_{55})_{i}\phi_{0j}-A_{44}\sum_{j=1}^{N}c_{ij}^{(1)}w_{0j}+\\
&\left\{A_{44}+(A_{44}\sum_{j=1}^{N}c_{ij}^{(1)}u_{j}^{1})-(B_{44}\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{j}^{1})+(A_{55}\sum_{j=1}^{N}c_{ij}^{(1)}w_{j}^{1})-(A_{55}\phi_{0i}^{1})-(T_{66})\right\}\phi_{i}=0\\
&+B_{11}\sum_{j=1}^{N}c_{ij}^{(1)}u_{0j}+B_{11}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}u_{j}^{1})_{i}u_{0j}-D_{11}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{j}^{1})_{i}u_{0j}+B_{44}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}w_{j}^{1})_{i}u_{0j}-\\
&B_{44}\sum_{j=1}^{N}c_{ij}^{(1)}(\phi_{i}^{1})_{i}u_{0j}-\sum_{j=1}^{N}c_{ij}^{(1)}(T_{55})_{i}u_{0j}-D_{11}\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{0j}-D_{11}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}u_{j}^{1})_{i}\phi_{0j}+H_{11}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{j}^{1})_{i}\phi_{0j}-\\
&D_{44}\sum_{j=1}^{N}c_{ij}^{(1)}(\sum_{j=1}^{N}c_{ij}^{(1)}w_{j}^{1})_{i}\phi_{0j}+D_{44}\sum_{j=1}^{N}c_{ij}^{(1)}(\phi_{j}^{1})_{i}\phi_{0j}+\sum_{j=1}^{N}c_{ij}^{(1)}(T_{77})_{i}\phi_{0j}+B_{44}\sum_{j=1}^{N}c_{ij}^{(1)}w_{0j}+\\
&\left\{-B_{44}-(B_{44}\sum_{j=1}^{N}c_{ij}^{(1)}u_{j}^{1})+(D_{44}\sum_{j=1}^{N}c_{ij}^{(1)}\phi_{0j}^{1})-(A_{66}\sum_{j=1}^{N}c_{ij}^{(1)}w_{j}^{1})+(A_{66}\phi_{i}^{1})-(T_{88})\right\}\phi_{i}=0
\end{aligned}
\tag{22}
$$

Which

$$
\begin{aligned}
T_{44}&=(A_{11}\alpha_{x}(z)+A_{22}\alpha_{z}(z)+2A_{44}\alpha_{xy}(z))\Delta T\\
T_{55}&=(B_{11}\alpha_{x}(z)+B_{22}\alpha_{z}(z)+2B_{44}\alpha_{xy}(z))\Delta T\\
T_{66}&=(A_{44}\alpha_{x}(z)+A_{33}\alpha_{z}(z)+2A_{55}\alpha_{xy}(z))\Delta T\\
T_{77}&=(D_{11}\alpha_{x}(z)+D_{22}\alpha_{z}(z)+2D_{44}\alpha_{xy}(z))\Delta T\\
T_{88}&=(B_{44}\alpha_{x}(z)+B_{33}\alpha_{z}(z)+2A_{66}\alpha_{xy}(z))\Delta T
\end{aligned}
\tag{23}
$$

The presented coefficients in Equations. (10 - 11) and (19- 23) introduced as Equations. (24)

$$
\begin{aligned}
&\begin{Bmatrix}A_{11}\\B_{11}\\D_{11}\\H_{11}\end{Bmatrix}=\int_{A}\bar{Q}_{11}(z)\begin{Bmatrix}1\\z\\z^{2}\\z^{3}\end{Bmatrix}dA,\quad \begin{Bmatrix}A_{55}\\A_{66}\end{Bmatrix}=\int_{A}\bar{Q}_{66}(z)\begin{Bmatrix}1\\z\end{Bmatrix}dA,\quad \begin{Bmatrix}A_{22}\\A_{33}\\A_{44}\end{Bmatrix}=\int_{A}\begin{Bmatrix}\bar{Q}_{12}(z)\\\bar{Q}_{26}(z)\\\bar{Q}_{16}(z)\end{Bmatrix}dA\\
&\begin{Bmatrix}B_{22}\\B_{33}\\B_{44}\end{Bmatrix}=\int_{A}z\begin{Bmatrix}\bar{Q}_{12}(z)\\\bar{Q}_{26}(z)\\\bar{Q}_{16}(z)\end{Bmatrix}dA,\quad \begin{Bmatrix}D_{22}\\D_{33}\\D_{44}\end{Bmatrix}=\int_{A}z^{2}\begin{Bmatrix}\bar{Q}_{12}(z)\\\bar{Q}_{26}(z)\\\bar{Q}_{16}(z)\end{Bmatrix}dA,\quad \begin{Bmatrix}I_{0}\\I_{1}\\I_{2}\end{Bmatrix}=\int_{A}\rho(z)\begin{Bmatrix}1\\z\\z^{2}\end{Bmatrix}dA
\end{aligned}
\tag{24}
$$

3. Results And Discussions

To validate the obtained Equations and considered numerical method, a comparison between the previous papers in this case performed.

The effective Young modulus of the shell assumed to vary as a power law of the thickness coordinate. Properties of considered ceramic, metal and function of FGM presented as:

$$E(z) = (E_m - E_c)(\frac{z}{h} + \frac{1}{2})^n + E_c$$
$$E_m = 70GPa, \quad v_m = 0.3 \qquad (25)$$
$$E_c = 380GPa, \quad v_c = 0.3$$

In Table. (1) the first and fifth frequencies of isotropic FGM beam, with simply-simply boundary condition in different length to thickness ratio, and various volume fraction powers, which obtained from the current method, compared the presented results with reference (Aydogdu and Taskin, 2007).

7. **Table 1** comparison between first and fifth non-dimensional frequencies of FG beam with simply-simply boundary condition $\Omega = \frac{\omega L^2}{h}\sqrt{\frac{\rho_m}{E_m}}$

L/h		Theory	Power law index (n)				
			0	0.1	1	2	10
5	Ω_1	[21]	6.563	6.237	4.652	4.101	3.563
		Present	6.5632	6.2372	4.6533	4.1025	3.5610
	Ω_5	[21]	91.163	87.019	65.946	57.423	46.716
		Present	91.1632	87.2006	66.7216	58.2700	47.2857
20	Ω_1	[21]	6.931	6.580	4.895	4.323	3.791
		Present	6.9313	6.5808	4.8950	4.3234	3.7914
	Ω_5	[21]	159.347	151.495	113.17	99.677	86.089
		Present	159.3449	151.5012	113.2002	99.7117	86.1200

The results inTables.1 are shown that there is a great likeness between the present and the previous obtained results.

Figure.1 illustrates the effect of fiber angle`s positioning inorthotropic material. In this Figure the non-dimensional first natural frequency for an one-layer orthotropic beam with a fibersorientation throughout the thickness of the beam is illustrated. In addition to affect of fibers angle, effect of temperature increases on the non-dimensional first natural frequency illustrated.

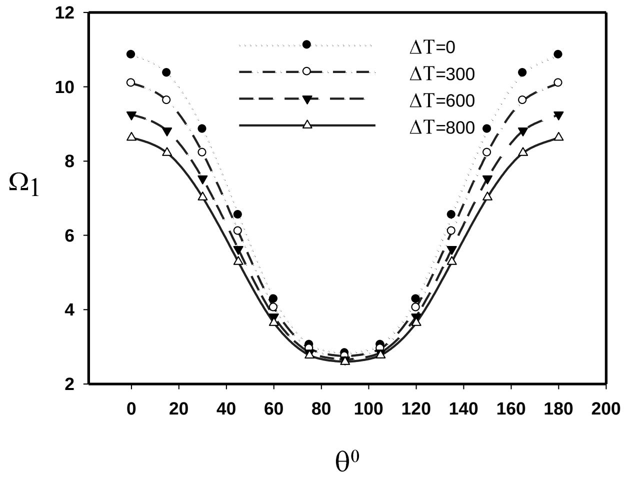

Figure 1variations of the non-dimensional first natural frequency versus one -layer homogenous orthotropic beam`s fibers angle. $\Delta T = 0$, L/h=10)

The results in Fig. 1 are shown that the non-dimensional first natural frequency of one-layer orthotropic beam decreases up to 90^0, which in this angle the minimum natural frequency viewed. On the other hand, the natural frequency in all boundary conditions and every temperature is symmetrical toward 90^0 angle. For example, the frequencies are equal in 60^0 and 120^0 or 40^0 and 140^0 because of being symmetric toward 90degrees angle.
Another point is the decrease of natural frequency in all governing boundary conditions because of temperature increase, whatever the fiber angle is closer to *90* degrees, the difference between the natural frequency of a determined angle in various temperatures will be lesser. The minimum difference between natural frequencies in different temperatures occurs at *90* degrees, and the maximum in *0* and *180* degrees.
Figure.2 showsthe variations of non-dimensional natural frequency in different powers from Equations. (1) and (2) for symmetric, asymmetric, and classic distributions of fibers orientationcontinuously in FOFG beam. The non-dimensional natural frequency increases by increasing the power low index. Symmetrical and asymmetrical distributions have the highest and the lowestnon-dimensional natural frequencies respectively in all boundary conditions. The results of three distributions are equal in higher ranges of power, and the variations of non-dimensional natural frequencies are infinitesimal toward power.

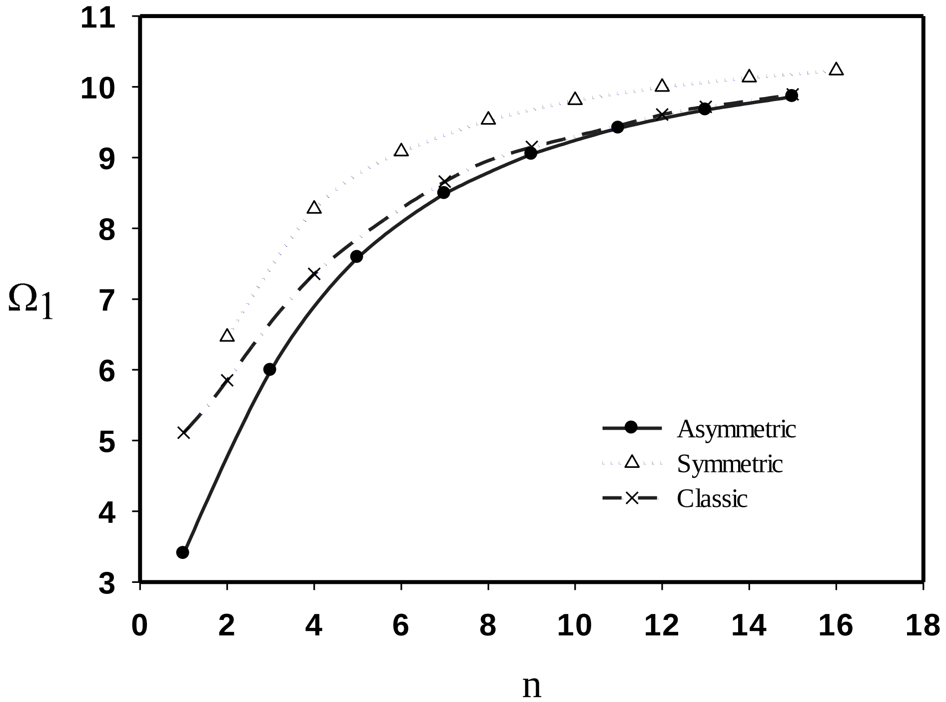

Figure 2. the first natural frequency`s variations versus power for FOFG beam with different fibers angle positioning distributions. ΔT = 0, L/h=10)

Figure.3 illustrates the variations of the non-dimensional first natural frequency in temperature increase for different distributions and boundary conditions of FOFG beam. Affects of dependence or independence of properties toward temperature is the case which studied more than the mentioned parameters. This Figure, showdecrease of natural frequency per increase of temperature in 0-800 K range.The non- dimensional natural frequency decrease is infinitesimal when the properties considered temperature independent,but there is a noticeable decrease in natural frequency when properties are temperature dependent. The difference in natural frequency in the cases where properties are temperature dependent or independent is lower and higher in lower and higher temperatures, respectfully. As like Figure.3, below Figureindicates the symmetrical and asymmetrical distributions have highest and lowest natural frequencies in- order.

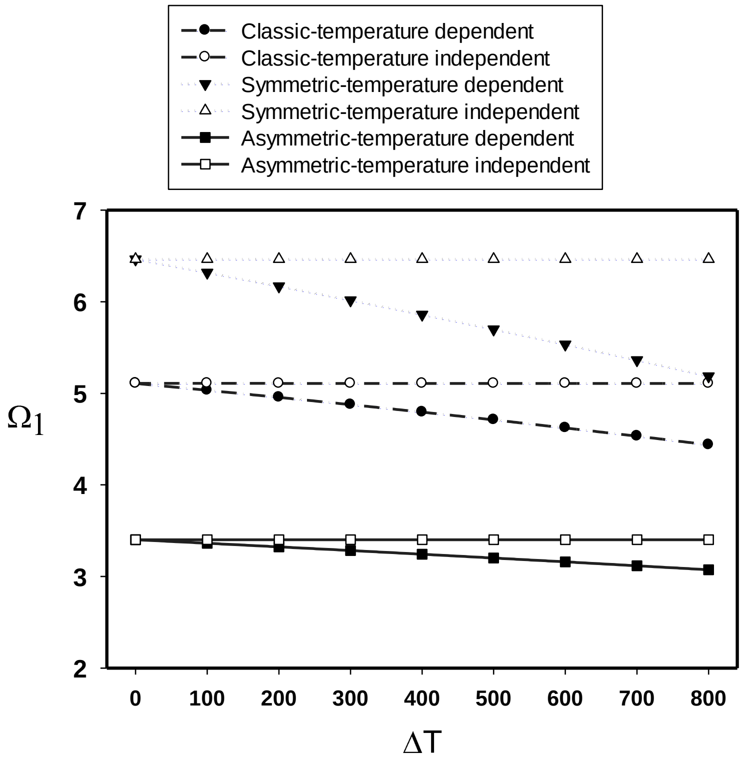

Figure 3. variations of non-dimensional first natural frequency versus smooth increase of temperature for different distributions of fibers orientation, and temperature dependent and independent properties. L/h=10

In Figure.4, variations of the non-dimensional first natural frequency in terms of length to thickness ratio per even powers which related to the symmetrical distribution and different boundary conditions are shown. In all boundary conditions, increase of length to thickness ratio and power leads to increase of natural frequency. Variations of natural frequency toward length to thickness ratio are impalpable for small values of constant powers which clearly observed in Figures .4. The variations are higher and more considerable by power increase.

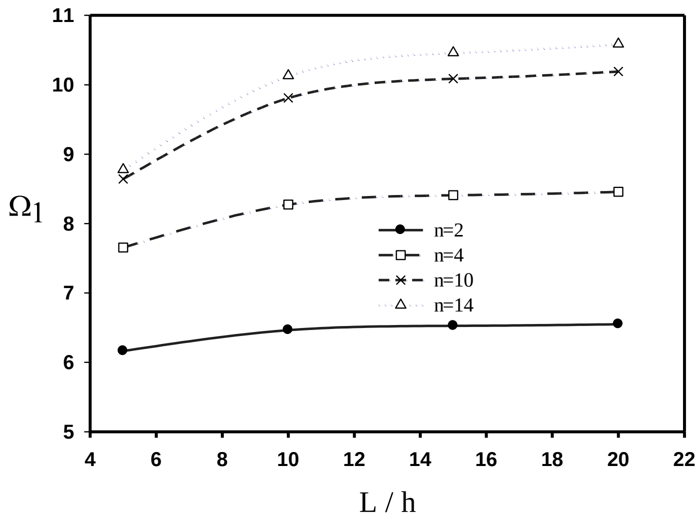

Figure 4. variations of the first natural frequency versus of length to thickness for FOFG beam per symmetrical distribution and different even powers$\Delta T=0$)

In Tables.2 first three non-dimensional natural frequencies of FOFG beam presented for three Classical, symmetrical, and asymmetrical distributions of fibers orientation in thickness direction per different temperatures. Although, the non-dimensional first natural frequency of FOFG beam presented for the two temperatures 200 and 4000K , distribution of fibers angle and powers low index,in order in Tables.3.

Table2 non-dimensional first three natural frequencies of FOFG beam with classical distribution (n=1) of fibers orientation, (L/h=20)

		$\Delta T(K)$								
BCs		0	100	200	300	400	500	600	700	800
classical	Ω_1	5.177108	5.100696	5.022359	4.941753	4.858732	4.773133	4.680049	4.591839	4.495434
	Ω_2	20.43593	20.13503	19.8267	19.50918	19.18231	18.84536	18.4874	18.13212	17.75313
	Ω_3	45.0179	44.35814	43.68166	42.98615	42.26998	41.53119	40.74492	39.96989	39.14026
symmetrical	Ω_1	6.548434	6.400772	6.249601	6.094707	5.935702	5.772214	5.60435	5.430985	5.251577
	Ω_2	25.85666	25.27473	24.67902	24.06861	23.44211	22.79813	22.13634	21.45335	20.74709
	Ω_3	56.98753	55.70971	54.40167	53.0614	51.68591	50.27216	48.81919	47.31992	45.76979
asymmetrical	Ω_1	3.422228	3.383858	3.343997	3.304891	3.264274	3.222658	3.181073	3.137141	3.095068
	Ω_2	13.60526	13.45137	13.29182	13.13446	12.9718	12.80563	12.63868	12.46448	12.29402
	Ω_3	30.30695	29.96097	29.60582	29.25113	28.88708	28.51573	28.14061	27.75257	27.3662

Table3 non-dimensional first natural frequencies of FOFG beam with different distributions of fibers orientation, (L/h=20, ΔT=200K)

Models	n	ΔT=400K	ΔT=200K
Symmetric	4	7.6145	8.0479
	6	8.4027	8.8907
	8	8.8553	9.3729
Asymmetric	3	5.6098	5.8493
	5	7.0815	7.4310
	7	7.9341	8.3499
Classic	2	5.5295	5.7450
	4	6.8962	7.2239
	7	8.1142	8.5429

4. CONCLUSION

In this paper, thermal vibration of the orthotropic beam reinforced withfibers investigated in which fibers angle changes functionally and continuously in thickness direction. Three models for variations of fibers angle along the thickness of the beam is considered. In this literature, the uniformly assumed temperature affect on the beam, different types of fibers angle and geometrical parameters investigated.The Equations ofmotion obtained and solved based on first order theory and differential quadrature method, respectively.

-Increase of power in all three classical, symmetrical and asymmetrical distributions leads to increase of non-dimensional natural frequency andthe natural frequency is constant and inclines to the natural frequency of orthotropic one - layer beam per higher powers

-Symmetrical and asymmetrical distributions have in-order the highest and the lowest frequency for the three considered distributions of fibers angle variation in thickness direction which the difference in natural frequencies for these three distributions is higher in lower powers, and it gets lower by increasing the powers.And finally,it inclines to the natural frequency of orthotropic one- layer beam.

The non-dimensional natural frequencies decrease by increasing the temperature throughout the FOFG beam,and the case gets closer to its real status when the properties of beam considered temperature dependent.

In temperature dependent properties manner, the further decrease occurs in natural frequencyand its values will be less than the temperature independent properties manner. Quantitative difference between the natural frequencies of two temperature dependent and independent properties state is greater in the higher temperatures. This is true about all boundary conditions and those three types of distribution fibers angle.
Increase of length to thickness ratio leads natural frequency`s increase which the greater constant powers create greater natural frequency modifications per variations of length to thickness ratio.

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