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A study of functionally graded porous beam based on simple beam theory

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ABSTRACT

In this article, the bending behaviors of functionally graded porous (FGP) beams are determined associated with uniform load. The simple beam theory is carried out with various boundary conditions. Two types of porosity are also applied to study the influences of material properties on bending behaviors. The results obtained in this article are presented and compared with other results in the references to verify the correctness in implementing the formula and writing the Matlab code. Last but not least, this article can help researchers to have an overview of the bending characteristics of the functionally graded porous beams.

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1. INTRODUCTION

In the last few decades, functionally graded material has become one of the smart materials and it is widely used in industry. The idea is to produce a smart material by changing the micro structure with a specific gradient from one material to another material. This enables the material to have the best behavior of both materials. If it is for thermal, or corrosive resistance or malleability and toughness, both strengths of the material may be used to avoid troubles related to above issues [1-5]. Due to the wide application of smart materials, various studies have been conducted on the mechanical behavior of structures as [6-14]. However, this material can exhibit some deficiencies such as porosity during the manufacturing process [15, 16]. So, for a good knowledge of porosity effect on mechanical behavior of FG structures, a study related to this issue must be added soon. Among three kinds of structure like beam, plate and shell, beam has always considered the interests of researchers because of its applications. There are many different beam theories used to analyze beam structures like simple beam theory [17, 18], classical beam theory [19, 20], first-order shear deformation theory [21, 22] or higher-order shear deformation theory [23-25]. However, using a simple model helps us to reduce the computational cost with the resulting error within the allowable range. Furthermore, beams made of functionally graded materials with existing porosity should be studied as much as possible to help the designer have a correct view of the mechanical properties. Indeed, the few published papers on bending static behavior of FGP beams are presented. Atmane *et al.* presented bending, free vibration and buckling responses of FGP beams resting on elastic foundations via an efficient quasi-3D theory [26]. Souhir et al. explore the influence of porosity on bending static analysis of functionally graded (FG) beams using a refined mixed finite element beam model [27], Jouneghani *et al.* [28] investigated the mechanical behavior of FGP nanobeams and subjected to a hygro-thermo-mechanical loading., etc. From above reasons, this article is given to investigate the bending behavior of functionally graded porous beams respectively.

This article has four sections. Sect. 1 gives the introduction as above. Sect. 2 presents the formulations as well as Sect. 3 shows some essential results. Finally, a few comments are also given in Sect. 4 respectively

2. FORMULATIONS

A FGP beam of length L, width b and thickness t is studied. It is made by continuously changing from ceramic to metal phases through the thickness direction. The volume fraction V_m of metal phase follows a power-law distribution which can be written as

$$V_m = \left(\frac{z}{h} + 0.5\right)^n \tag{1}$$

$$V_m + V_c = 1 \tag{2}$$

where z is the coordinate in the thickness direction and n is called as the power-law index.
Due to actual problems in the fabrication process, porosities may appear as an imperfection in the FG beams leading hence to two types of porosity, namely even and uneven distributions as shown in Figure 1. The effective material properties of the FGP beam are determined using the modified rule of mixture in which the porosity volume fraction, α, 0≤α<1, affects averagely the material volume fraction of each constituent

$$P(z,\alpha) = (P_m - P_c)V_m + P_c - \frac{\alpha}{2}(P_m - P_c) \quad \text{even porosity} \tag{3}$$

$$P(z,\alpha) = (P_m - P_c)V_m + P_c - \frac{\alpha}{2}(P_m - P_c)\left(1 - 2\frac{|z|}{h}\right) \quad \text{uneven porosity} \tag{4}$$

It can be mentioned that for even-type the porosity phases are uniformly distributed along the beam cross section whereas for uneven-type, the porosity phases are spreading mostly around the middle surface of the beam cross section and vanish in the top and bottom surfaces.

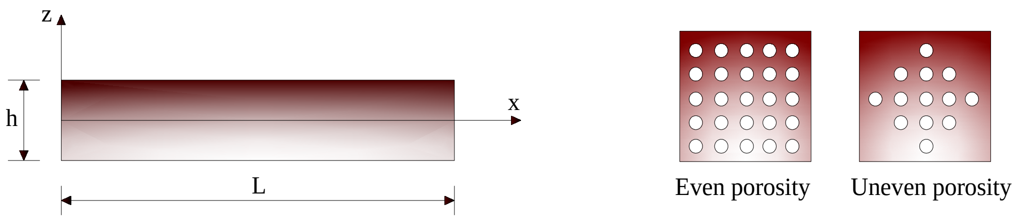

Figure 1. Functionally graded porous beam with two types of porosity

Based on finite element method (FEM), the degrees of freedom associated with a node of a beam element are a transverse displacement and a rotation as depicted in Figure 2. This transverse displacement can be written

$$v_e = N_{v_i} v_i + N_{\theta_i}\theta_i + N_{v_j} v_j + N_{\theta_j}\theta_j \tag{5}$$

in which

$$N_{v_i} = 1 - \frac{3x^2}{L_e^2} + \frac{2x^3}{L_e^3} \tag{6}$$

$$N_{\theta_i} = x - \frac{2x^2}{L_e} + \frac{x^3}{L_e^2} \tag{7}$$

$$N_{v_j} = \frac{3x^2}{L_e^2} - \frac{2x^3}{L_e^3} \tag{8}$$

$$N_{\theta_j} = -\frac{x^2}{L_e} + \frac{x^3}{L_e^2} \tag{9}$$

Using the principles of simple beam theory, the beam element stiffness matrix will be derived

$$K_e = \frac{E_e I_e}{L_e^3}\begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix} \tag{10}$$

According to the principle of minimum total potential energy, the element equation can be described as

$$\frac{E_e I_e}{L_e^3}\begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix}\begin{Bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{Bmatrix} = \begin{Bmatrix} f_i \\ m_i \\ f_j \\ m_j \end{Bmatrix} \tag{11}$$

After assembly, the bending parameters can be obtained by solving the following equation

$$\mathbf{Kd} = \mathbf{F} \tag{12}$$

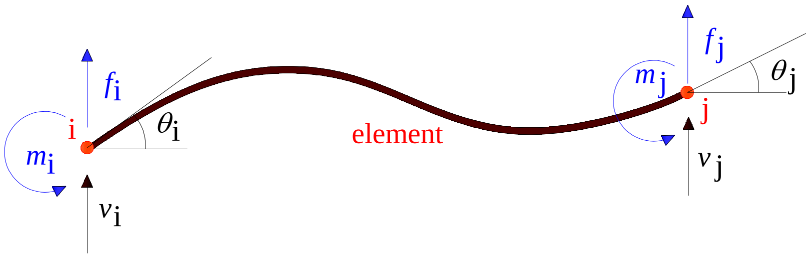

Figure 2. The simple beam element

Four boundary conditions can also be listed here with subscripts ‘C’, ‘S’ and ‘F’ refer to the clamped, simply supported and free condition respectively

(SS) $$v(0) = v(\mathrm{L}) = 0 \tag{13}$$

(CS) $$v(0) = \theta(0) = 0,\ \ v(\mathrm{L}) = 0 \tag{14}$$

(CC) $$v(0) = \theta(0) = 0,\ \ v(\mathrm{L}) = \theta(\mathrm{L}) = 0 \tag{15}$$

(CF) $$v(0) = \theta(0) = 0 \tag{16}$$

To be more specific, the finite element system of equations can be reached as below

- Input data
  - Geometric data and material properties
- Calculating constitutive matrix
- Loop over elements
  - Calculating element stiffness matrix
  - Calculating element force vector
- Assembling the element stiffness matrix and force vector in the global coordinate system
- Applying boundary conditions
- Solving equation for bending analysis
- Display deflections and rotations at nodes.

3. RESULTS AND DISCUSSIONS

In this section, as a first step, the validity of the proposed model is checked for (SS) FGP beams under a uniform load q = 10^6 N/m^2. The structure is made of Aluminium/Alumina composite (Al/Al2O3) with the following material properties as in Table 1. The transverse displacement at position L/2 can be normalized by $\bar{v} = 100\frac{\mathrm{E_m t^3}}{\mathrm{qL^4}} v(\frac{\mathrm{L}}{2})$. The values of above parameters for FGP beams with L/t = 5, three values of porosity coefficient α and n=2 are presented in Table 2 and compared with others beam theories by Atmane *at al.* [26] or Souhir *et al.* [27].

Table 1. The material properties

(Al_2O_3 / Al)
E_c = 380 x 10^9 Pa, ν_c = 0.3,
ρ_c = 3960 kg/m^3,
E_m = 70 x 10^9 Pa, ν_m = 0.3,
ρ_m = 2702 kg/m^3

Table 2. The comparison of the normalized transverse displacements at position x= L/2 of (SS) FGP beams with L/t = 5, n=2 and α = 0, 0.1 & 0.2

α	Souhir *et al.* [27]	Atmane *et al.* [26]	Present
0	5.20	5.35	5.35
0.1	5.82	6.22	6.06
0.2	6.63	7.38	6.98

A good agreement is disclosed between the results which indicates the feasibility of the present model in the prediction of bending behavior of FGP beams. The relative error among above results can be explained by the different beam theories used in all studies.

In the next step, the influences on the transverse displacement $V = v(x = 0 \rightarrow L)$ and rotation angle θ along the length of FGP beams from the change of boundary conditions CC, CS & CF with two types of porosity are depicted in Figures 3-8.

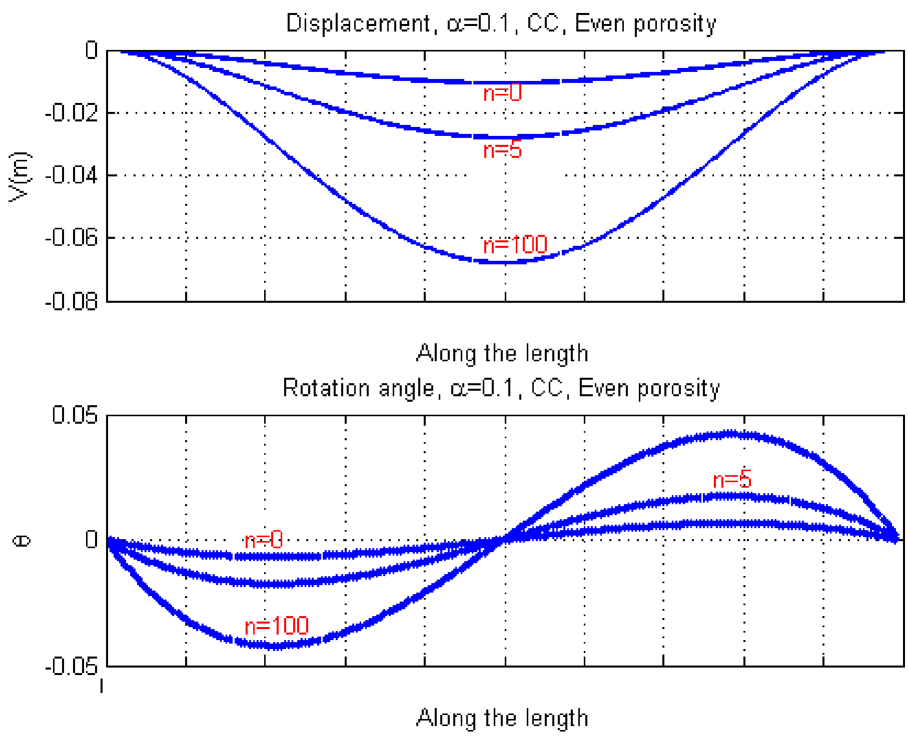

Figure 3. The change of transverse displacement and rotation along the length of FGP (CC) beams with α = 0.1, even porosity and n = 0, 5 & 100.

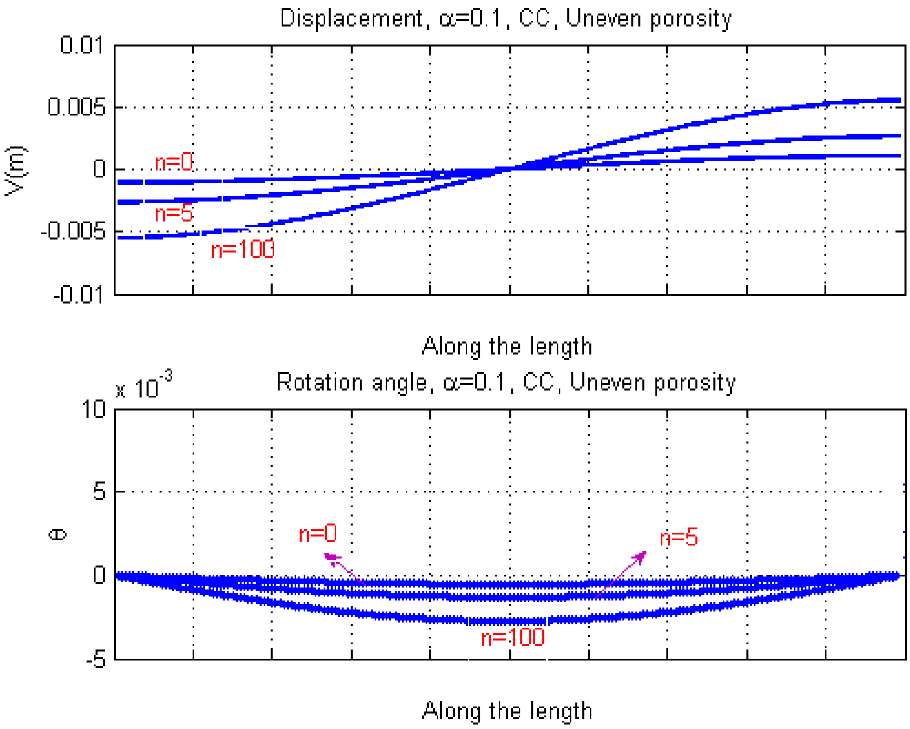

Figure 4. The change of transverse displacement and rotation along the length of FGP (CC) beams with α = 0.1, uneven porosity and n = 0, 5 & 100.

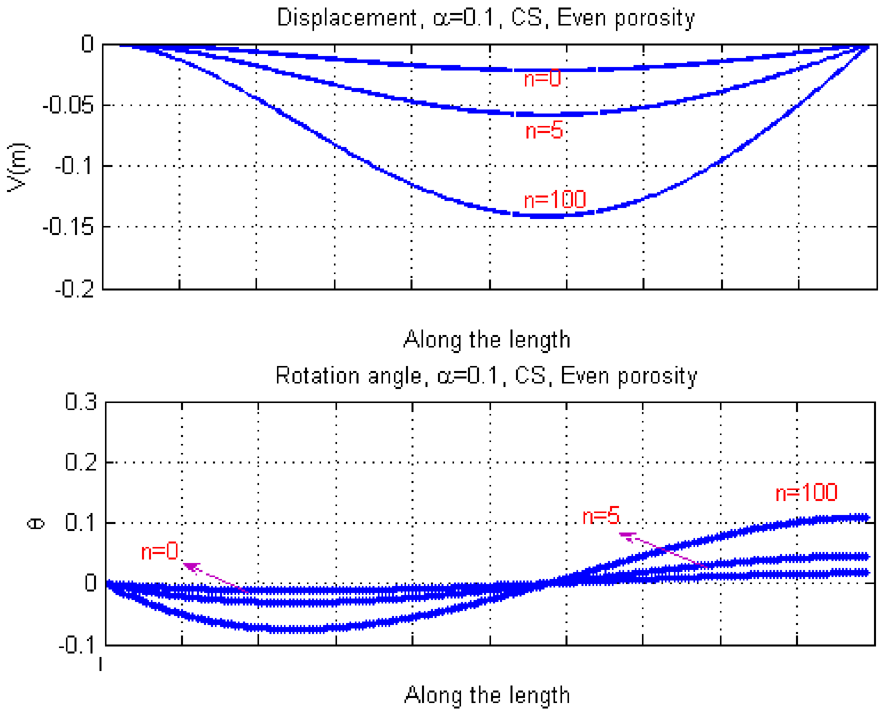

Figure 5. The change of transverse displacement and rotation along the length of FGP (CS) beams with α = 0.1, even porosity and n = 0, 5 & 100.

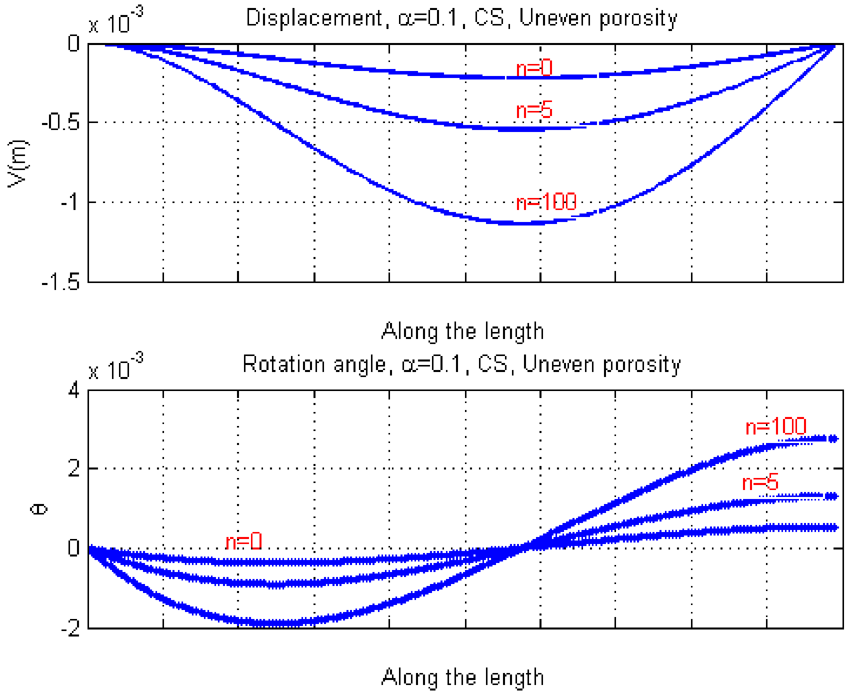

Figure 6. The change of transverse displacement and rotation along the length of FGP (CS) beams with α = 0.1, uneven porosity and n = 0, 5 & 100.

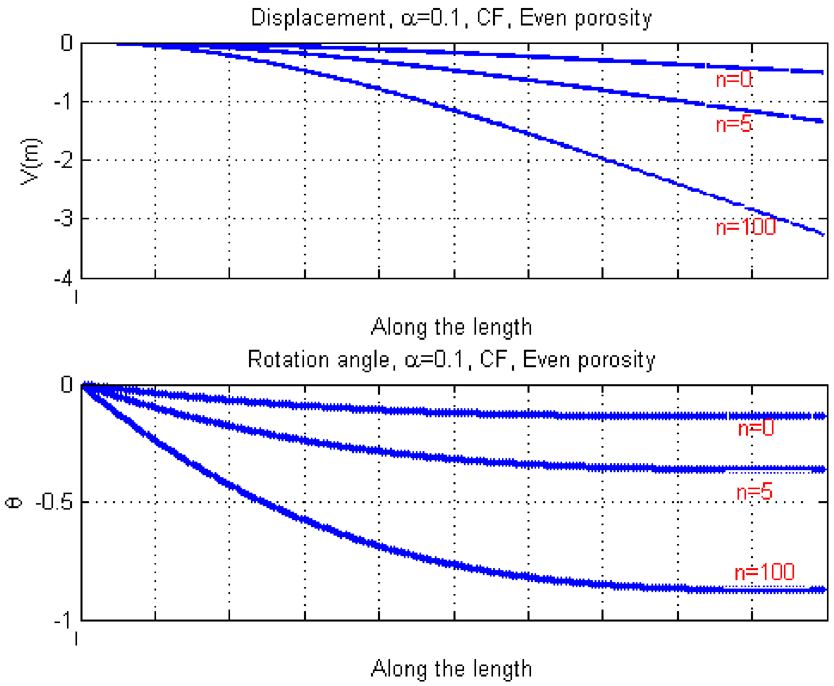

Figure 7. The change of transverse displacement and rotation along the length of FGP (CF) beams with α = 0.1, even porosity and n = 0, 5 & 100.

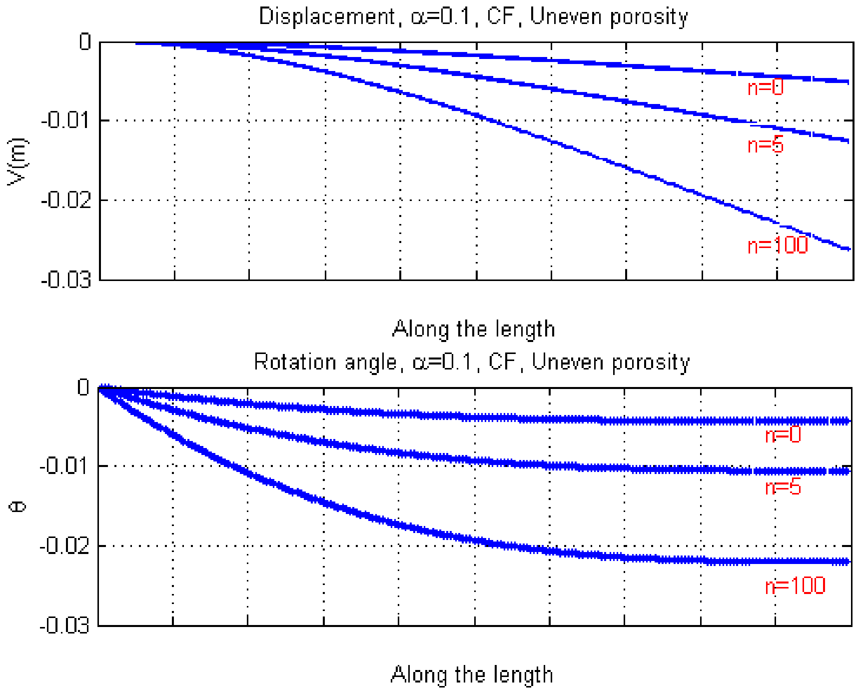

Figure 8. The change of transverse displacement and rotation along the length of FGP (CF) beams with α = 0.1, uneven porosity and n = 0, 5 & 100.

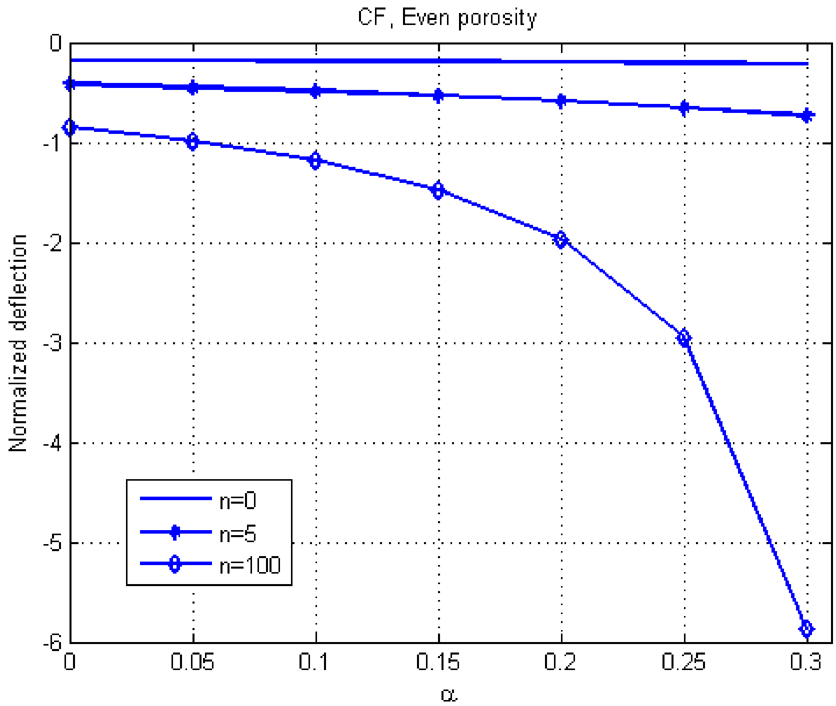

Figure 9. The normalized transverse displacement of FGP (CF) beams with various values of α, even porosity and n = 0, 5 & 100.

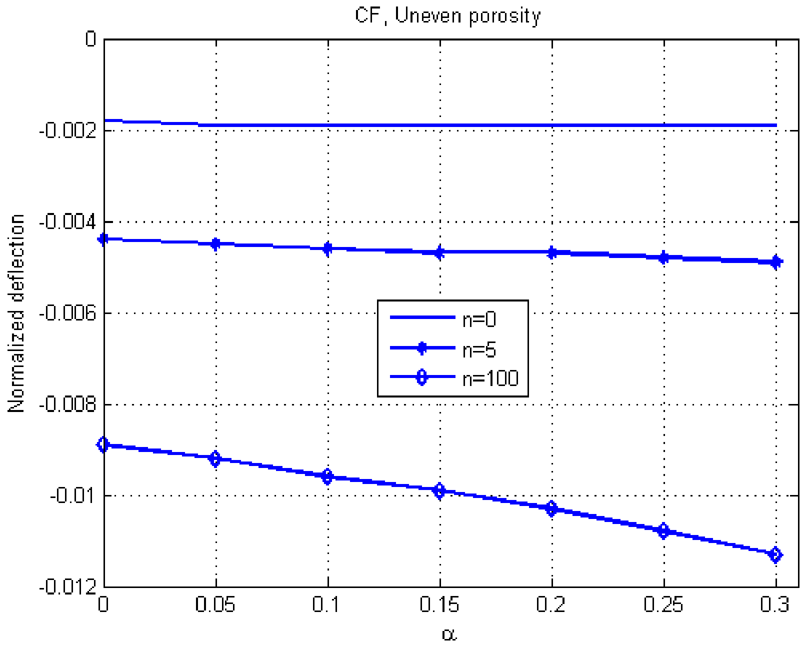

Figure 10. The normalized transverse displacement of FGP (CF) beams with various values of α, uneven porosity and n = 0, 5 & 100.

With n = 0, the deflection and rotation angle of FGP beams are the smallest for all cases. This can be explained by the full ceramic properties of the material. Moreover, by changing the porosity coefficient α from 0 to 0.3, the results of the normalized transverse displacement $\overline{v} = \frac{1}{t}v(L/2)$ of FGP beams with (CF) boundary condition are plotted in Figure 9 and 10 for two even and uneven types of porosity. As the porosity value increases, the deflection of FGP beam also increases and this statement holds for all values of n, respectively.

4. CONCLUSION

In this work, author presents the bending behaviors of functionally graded porous (FGP) beams under four different types of boundary condition and two kinds of porosity. The results of this paper are good, agree well with others in references. Although the topic and approach of the paper are not new, the main aim of the author is to affirm the applicability of the simple beam theory to analyze the functionally graded porous (FGP) beams with acceptable results

REFERENCES

[1] O. Carvalho, M. Buciumeanu, G. Miranda, S. Madeira, and F. S. Silva, "Development of a method to produce FGMs by controlling the reinforcement distribution," *Materials & Design*, vol. 92, pp. 233-239, 2016.

[2] C.-Y. Huang and Y.-L. Chen, "Effect of mechanical properties on the ballistic resistance capability of Al2O3-ZrO2 functionally graded materials," *Ceramics International*, vol. 42, no. 11, pp. 12946-12955, 2016.

[3] M. Naebe and K. Shirvanimoghaddam, "Functionally graded materials: A review of fabrication and properties," *Applied Materials Today*, vol. 5, pp. 223-245, 2016.

[4] G. Udupa, S. S. Rao, and K. V. Gangadharan, "Functionally Graded Composite Materials: An Overview," *Procedia Materials Science*, vol. 5, pp. 1291-1299, 2014.

[5] H. L. Ton-That, H. Nguyen-Van, and T. Chau-Dinh, "A novel quadrilateral element for analysis of functionally graded porous plates/shells reinforced by graphene platelets," *Archive of Applied Mechanics*, vol. 91, no. 6, pp. 2435-2466, 2021.

[6] T. Van Do, D. K. Nguyen, N. D. Duc, D. H. Doan, and T. Q. Bui, "Analysis of bi-directional functionally graded plates by FEM and a new third-order shear deformation plate theory," *Thin-Walled Structures*, vol. 119, pp. 687-699, 2017.

[7] T.-T. Hoang Lan, "A Combined Strain Element to Functionally Graded Structures in Thermal Environment," *Acta Polytechnica*, vol. 60, no. 6, pp. 528-539, 2020.

[8] S. P. Parida, P. C. Jena, and R. R. Dash, "FGM Beam analysis in Dynamical and Thermal surroundings using Finite Element Method," *Materials Today: Proceedings*, vol. 18, pp. 3676-3682, 2019.

[9] Y. Chen, G. Jin, C. Zhang, T. Ye, and Y. Xue, "Thermal vibration of FGM beams with general boundary conditions using a higher-order shear deformation theory," *Composites Part B: Engineering,* vol. 153, pp. 376-386, 2018.
[10] F. Ebrahimi and E. Salari, "Thermal buckling and free vibration analysis of size dependent Timoshenko FG nanobeams in thermal environments," *Composite Structures,* vol. 128, pp. 363-380, 2015.
[11] H. L. Ton-That, "Improvement on eight-node quadrilateral element (IQ8) using twice-interpolation strategy for linear elastic fracture mechanics," *Engineering Solid Mechanics,* vol. 8, no. 4, pp. 323-336, 2020.
[12] H. L. Ton-That, "The Linear and Nonlinear Bending Analyses of Functionally Graded Carbon Nanotube-Reinforced Composite Plates Based on the Novel Four-Node Quadrilateral Element," *European Journal of Computational Mechanics,* vol. 29, no. 1, pp. 139-172, 2020.
[13] T.-T. Hoang Lan and H. Nguyen-Van, "A Combined Strain Element in Static, Frequency and Buckling Analyses of Laminated Composite Plates and Shells," *Periodica Polytechnica Civil Engineering,* vol. 65, no. 1, pp. 56-71, 2021.
[14] D.-Q. Vo and H. L. Ton-That, "Free vibration of simply supported steel I-girders with trapezoidal web corrugations," *Reports in Mechanical Engineering,* vol. 1, no. 1, pp. 141-150, 2020.
[15] A. Nawaz and S. Rani, "Fabrication and evaluation of percent porosity and density reduction of aluminium alloy foam," *Materials Today: Proceedings,* 2021.
[16] M. H. Ghayesh and A. Farajpour, "A review on the mechanics of functionally graded nanoscale and microscale structures," *International Journal of Engineering Science,* vol. 137, pp. 8-36, 2019.
[17] S. R. Singiresu, *The Finite Element Method in Engineering*, Sixth ed. Elsevier 2018.
[18] K. H. Yang, *Basic Finite Element Method as Applied to Injury Biomechanics*, First ed. Elsevier, 2018.
[19] S. Yin, Z. Xiao, Y. Deng, G. Zhang, J. Liu, and S. Gu, "Isogeometric analysis of size-dependent Bernoulli–Euler beam based on a reformulated strain gradient elasticity theory," *Computers & Structures,* vol. 253, p. 106577, 2021.
[20] J. W. Lee and J. Y. Lee, "Free vibration analysis of functionally graded Bernoulli-Euler beams using an exact transfer matrix expression," *International Journal of Mechanical Sciences,* vol. 122, pp. 1-17, 2017.
[21] I. Katili, T. Syahril, and A. M. Katili, "Static and free vibration analysis of FGM beam based on unified and integrated of Timoshenko's theory," *Composite Structures,* vol. 242, p. 112130, 2020.
[22] P. Sharma and R. Singh, "A numerical study on free vibration analysis of axial FGM beam," *Materials Today: Proceedings,* vol. 44, pp. 1664-1668, 2021.
[23] H. Ma, "Rational approach for higher-order shear deformation beam theories," *Composite Structures,* vol. 251, p. 112599, 2020.
[24] P. V. Avhad and A. S. Sayyad, "Static analysis of functionally graded composite beams curved in elevation using higher order shear and normal deformation theory," *Materials Today: Proceedings,* vol. 21, pp. 1195-1199, 2020.
[25] Y. L. Pei, P. S. Geng, and L. X. Li, "A modified higher-order theory for FG beams," *European Journal of Mechanics - A/Solids,* vol. 72, pp. 186-197, 2018.
[26] H. A. Atmane, A. Tounsi, and F. Bernard., "Effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations," *International Journal of Mechanics and Materials in Design,* vol. 13, no. 1, pp. 71-84, 2017.
[27] S. Zghal, D. Ataoui, and F. Dammak, "Static bending analysis of beams made of functionally graded porous materials," *Mechanics Based Design of Structures and Machines,* pp. 1-18, 2020.
[28] F. Z. Jouneghani, R. Dimitri, and F. Tornabene, "Structural response of porous FG nanobeams under hygro-thermo-mechanical loadings," *Composites Part B: Engineering,* vol. 152, pp. 71-78, 2018.