International Journal of Engineering and Applied Physics (IJEAP)
Vol. 6, No.1, January 2026, pp. 1273~1284
ISSN: 2737-8071

DYNAMICS OF 2-D PRISMATIC TANKS WITH EXTENDED GEOMETRY

Kumar A.[1], Sinhamahapatra K.P.[2]
[1,2] Department of Aerospace Engineering, IIT Kharagpur-721302, West Bengal

Article Info

Article history:

Received July 20, 2025
Revised Nov 21, 2025
Accepted Jan 11, 2026

Keywords:

Sloshing
Analytical
Superposition
Undamped
Frequency response

ABSTRACT

Dynamic charectersitcs of fluid within a tank is dependent upon it's geometry. Undertstanding the variation of dynamics of composite shaped tanks with respect to the basic geometry is very important to decide suitable modification in geometry to meet the specifc mission objectives. Degree of precision, economy as well as suitability to the tank geometry are deciding factors in the selction of the dynamic analysis method. Computational methods offer wider applicability for ranges of shapes and are economical for repeated use. In the present analysis, the effects of geometrical modifications on the dynamic characteristic of the basic tank geometry have been studied assuming inviscid fluid in a tank applying FEM using triangular discretisation elements. Triangular discretization scheme has been adopted to resolve the singularity at a re-entrant angle in tanks of composite geometry and for closer approximation of the true geometry. A simple analytical method is also developed to compute the free surface reseponse near the fundamental mode and to evaluate the influence of the higher modes in the fluid sloshing in the modified composite tanks. Analytical results are compared with computational results for various types of tanks. Computed frequency response curves are used to decide the most suitable modification in terms of suppression of sloshing amplitude.

This is an open access article under the CC BY license.

Corresponding Author:

Kumar A.
Department of Aerospace Engineering IIT Kharagpur
Email: arunkumar_engr@yahoo.com

1. INTRODUCTION

Dynamic charecteristic of a fluid within a tank depends upon tank geometry. Tanks of more complex shapes are required to meet specifc system requirement and perfomance optimisation. Simple geometric tanks can be modified to a complex shape by truncation, extension as well as rotation. Due to the presence of re-entrant angles, determination of natural and forced dynamic responses of a composite shaped container becomes a challenging task.

Analytical methods to determine natural frequency of a rectangular tank was suggested by Abramson[1]. This analytical result is still continued to be used for validation of most of the numerical models. Elaborate analytical investigation on natural frequencies of different shaped tanks has been carried out by applying velocity potential over boundary surfaces of various composite shaped containers [2,3]. Analytical solution of triangular tanks has also been obtained assuming it as spectral solution [3]. Faltinsen and Timokha[4] used an approximate Trefftz solution for calculation of natural frequency of different 2-D composite shaped tanks like triangular, star-shaped, trapezoidal and prismatic tanks. Free surface response was not discussed.

Various researchers applied FEM with quadratic shape function to determine natural frequency of rectangular, trapezoidal and horizontal circular tanks using either a velocity potential formulation or a pressure based formulation. Results from forced dynamic analysis to compute free surface response under sinusoidal as well as earthquake excitations were compared with available experimental and numerical results for these tanks[5,6,7,8.9.10]. Boundry Element Method (BEM) as well as Commercial package ANSYS has also been used to determine natural frequency of arbitrary, truncated prismatic and various other shaped tanks[11,12].

Liquid sloshing in LNG carriers with 3D wedge shaped tank has been studied by applying Boundary Element Method (BEM) and a numerical approach has been developed to predict resonance condition[13,14]. Study was further extended on the topic by investigating nonlinear resonance condition in wedge shaped tanks [15]. Triangular discretisation technique has been utilised to analyse natural frequency of different type of 2D prismatic tanks and slosh suppression [16].

Experimental numerical and mechanical analog method has ben used to study effect of shape and geometric parameter on natural frequency, damping and behavior during strong and higher mode excitation in composite shaped tanks and bulk head tanks [17,18].

The survey on the existing literature shows that most of the studies are devoted to compute the natural frequency of prismatic and similar type of tanks but studies on variation in dynamics of tank due to change in geometrical shapes are extremely limited. Considerable research work has been done in computational field for analysis of forced response but there are very limitied efforts to study forced responses analyticaly. An analytical approach can be very useful for performance analysis of control measures

In this paper, effect on dynamics due to changes in geometry and forced dynamic analysis with periodic disturbances using FEM based on pressure formularion is carried out to determine the free surface response in these tanks. A method to compute an approximate forced response analyticaly considering only the fundamental mode for various elementary and composite shaped tanks is also developed which is used to anlyse the variation of maximum free surface response. A frequency response curve is also developed analytically that shows the influence of the fundamental mode.

2. RESEARCH METHOD

A pressure formulation based finite element method has been applied to study variation in natural dynamic characteristics of containers due to change in geometry of the basic tanks through geometrical extension. Triangular elements are used to discretize the fluid domain to ensure correct representation of the domain near the re-entrant angles. Lateral sloshing Type equation here.of fluid within a tank can be mathematically modelled as pendulum or a mass spring system. General motion of such system subject to harmonic excitation can be represented by a 2nd order differential equation.

$$M\ddot{y} + \overline{C}\dot{y} + Ky = F_p(t) \tag{1}$$

The solution of the above differential equation is sum of the general solution of the homogenous part of the equation which gives the eigenvalues or the natural frequencies of the system represented by the equation and the particular solution of the inhomogenous form represented by Eq (1). The general solution of the homogenous part can be written as

$$y_g(t) = A\cos\omega_n t + B\sin\omega_n t \tag{2}$$

Analytical solution is obtained for the nth mode natural frequency of a rectangular tank from Eq (2) as computed by Abramson [1]

$$\omega_n = \sqrt{g\kappa_n \tanh \kappa_n h} \tag{3}$$

Where κ_n is nth mode wave number and is equal to $n\pi/d$ with "*d*" being the width of the container. Analytical solution for other geometries can also be obtained. Details of such solution is available in Ibrahim[2] for wedge shaped canal, converging and diverging tanks. Construction of Eq (1) and obtaining the solution analytically is not straightforward for all type of systems, especialy when the system is repersenting a fluid domain with complex geometry. In the present work a pressure formulation based finite element numerical method with triangular elements is used to solve problem of oscillation of a fluid system in the complex composite shaped containers.

2.1 Pressure formulation and finite element analysis

Solutions for free surface problem can be approached by pressure (P) as primary variable. This approach helps in direct calculation of pressure at the nodal points, which is an important design parameter in structural problems. In this approach, basic equation is derived by application of the conservation laws on a fluid of density 'ρ' contained in a tank subjected to acceleration ($\ddot{U}$) in X, Y and Z directions of frame of reference. For an incompressible flow the equations, in terms of excess pressure, reduces to

$$\nabla^2 P = 0 \tag{4}$$

Above equation is solved subject to the following boundary conditions

a) Over free surface S_f, $P = 0$

b) Over bottom surface B_f, $\frac{\partial P}{\partial n} = 0$

c) At wall interface S_I, $\frac{\partial P}{\partial n} = -\rho \ddot{U}$

In addition, the kinematic boundary condition at the free surface needs to be satisfied. Assuming linear characteristics of free surface, the excess pressure at the free surface S_f is given by $P = \rho g \eta$. Differentiating twice with respect to time and using the dynamical equation of inviscid flow, the kinematic free surface condition can be obtained as

$$\rho \frac{\partial^2 \eta}{\partial t^2} = \frac{1}{g}\left(\frac{\partial P}{\partial y}\right)$$

Where η, free surface displacement (m) and g accelearation due to gravity (m/s^2). Since qnalytical solution is available for only some well-known simple geometry, which very rarely come in general practice, therefore numerical solution is preferred for complex geometry.

2.2.1 Solution of homogeneous equation

The homogeneous equation for natural dynamics is

$$M_f[\ddot{P}] + K_f[P] = \{0\} \quad (5)$$

Assuming a solution of the form $P = ae^{i\omega t}$ and substituting in Eq. (5) gives

$$\omega^2 = \left[M_f\right]^{-1} K_f \quad (6)$$

In an incompressible flow, mass matrix M_f is zero except for elements at the free surface. Therefore, computation can be economised by eliminating null elements of the matrix applying the well-known Guyan's condensation technique.

2.2.2 Solution of inhomogeneous equation

An externally excited system is represented by Eq. (5) with the force $F_p \neq 0$. The numerical solution can be obtained by an integration scheme, such as Newmark time integration scheme which is adopted in this work.This method is based on assuming an initial solution matrix and computing time derivative based on numerical finite difference technique suggested by Newmark.

$$\dot{P}_{t+\Delta t} = \dot{P}_t + a_6 \ddot{P}_t + a_7 \ddot{P}_{t+\Delta t} \quad (7)$$

$$\eta_t = \frac{P}{(\rho g)} \quad (8)$$

a_0 to a_7 are constants expressed as functions of two other constants α and β, as proposed by Newmark, and are available in numerous texts on numerical methods. Values of α and β in the present computations are chosen respectively as 0.5 and 0.25. A detail discussion on solution along with isoparametric shape function of descritizing elementry shape can be found in reference [5,6,10,16,19,20]

2.2.3 Effect of higher frequency

The particular solution can be determined by simple method of undetermined coefficients subject to the boundry conditions of the system by assuming a solution with some unknown coefficients like

$$y_p(t) = a\cos\omega t + b\sin\omega t \quad (9)$$

A detail solution can be found in Engineering Mathematics books dealing with solution to differential equation and any article on sinusoidal wave interaction [22,23]. Substituting the values of 1st and 2nd order derivatives in Eq (1) and equating coefficient of the sine and cosine terms on both sides, the values of "a" and "b" can be determined and the particular solution can be computed.

For an undamped system the solution can then be written *as*

$$y(t)=\frac{F_o}{M\left(\omega^2{}_n-\omega^2\right)}\left(\cos\omega t-\cos\omega_n t\right) \tag{10}$$

Eq (5) can be rearranged in following form

$$y(t)=\frac{2F_o}{M\left(\omega^2{}_n-\omega^2\right)}\sin\left(\frac{\omega_n+\omega}{2}\right)t\sin\left(\frac{\omega_n-\omega}{2}\right)t \tag{11}$$

This response repsersents superposition of two harmonic oscillations, oscillation of the undamped system with natural frequency ω_n and oscillation at the excitation frequency ω . The genearal solution for the undamped system at resonance can be written as

$$y(t)=\frac{F_o}{2m\omega_n}(1+t)\sin\omega_n t \tag{12}$$

The equation (11) and (12) provides the free surface displacement only if the fundamental mode is excited. An estimate of the influences of the higher modes can be obtained by comparing the results with those due to a numerical solution or an experimental measurement.

3. RESULTS AND DISCUSSIONS

3.1.1 Fluids in elementary shaped tanks

A rigid rectangular (R) tank of bottom width *"2a"* and liquid depth "*h"* is considered for analysis of natural frequencies of the contained liquid. Both rectangular and triangular elements are used for this case.

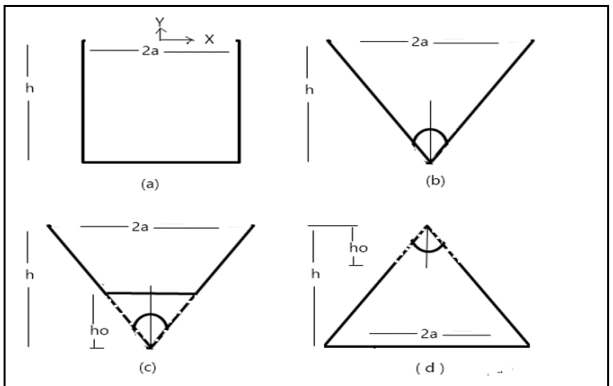


Fig (1) Elementary shaped tanks a) Rectangular (R) b) Diverging (D) c) Truncated Diverging (TD) d) Converging

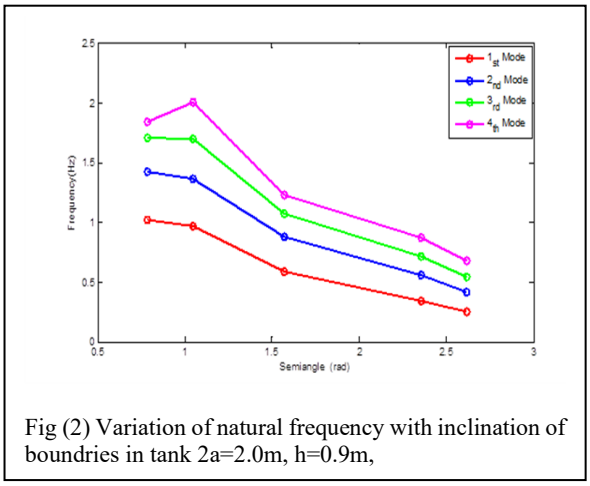


Fig (2) Variation of natural frequency with inclination of boundries in tank 2a=2.0m, h=0.9m,

A rectangular mesh consisting of 25×20 elements and a triangular mesh with 2×20×20 elements are considered for discretisation of the liquid domain. For validation purpose, computation has been compared with computed results from Pal [5] and analytical results from-NASA [1]. The comparison is presented in Table (1).. Computed results are in good agreement with previously computed values. Similarly natural frequency has been computed for an elementary diverging (D) tank shown in Fig (1b) with dimension *h*=1m and semiangle $\alpha=\frac{\pi}{4}$ and compared with the results from Zhe et al.[24] computed using Boundry Element Method as well as results from theoretical equation persented in Ibrahim[2] . Results are presented in Table (2). It is observed that accuracy of the present numerical computation for the first six modes is within 2% of the theoretical results. Level of accuracy for the first four modes is within 1%. Reason for higher degree of accuracy lies in the fact that the use of triangular descretization element allows capturing of the accurate boundary of tanks with high precision

Table (1): Natural frequency in a Rectangular tank (h=1.0m, $2a$=1.0m)

Mode	NASA [1]	Pal[5]	Present Computation	
			Rectangular	Triangular
1	0.88	0.88	0.8812	0.8828
2	1.24	1.26	1.2424	1.2545
3	1.53	1.54	1.5106	1.5436
4	1.76	1.80	1.7267	1.7930

Table (2) : Natural frequency in a diverging (D) tank (h=1m,α=π/4)

Mode	Theoretical	Zhe et al [24]	Present Numerical	%error
1	0.4989	0.5039	0.4985	-0.08
2	0.7601	0.7701	0.7603	0.026
3	0.9885	1.0030	0.9918	0.333
4	1.1684	1.1890	1.1782	-0.433
5	1.3259	1.3523	1.3432	1. 30
6	1.4660	1.4989	1.4938	1.89

Natural frequency of truncated diverging (TD) tank as shown in Fig (1c) with dimension h=1.0m and ho=0.4m, α=$\frac{\pi}{4}$ has also been numerically computed and presented in Table (3). The table also shows convergence in computed results with triangular elements N=240 and N=960. Computed results are compared with theoretical results from Ibrahim [2] and the results computed using Boundry Element Method (BEM) by Zhe et al. [24] . The table shows that the computed second and consecutive higher mode frequencies are comparable with the analytical and BEM frequencies. The lowest frequency obtained from the present computation is not seen in the referred works.

Table (3) Natural frequency (Hz) in a truncated diverging tank (h=1.2m, h_0=0.2m, $\alpha = \frac{\pi}{4}$)

Mode	Theoretical		Numerical		R tank (0.4mx1m)	D tank h=1m, $\alpha = \frac{\pi}{4}$
	Zhe et al. [24]	Ibrahim [2]				
			N=240	N=960		
1	0.68243	0.7049	0.4983	0.4973	1.4061	0.7050
2	0.971055	0.9969	0.7834	0.7702	2.0225	1.0752
3	1.201356	1.2210	0.993	0.9875	2.5319	1.4027
4	1.415706	1.4099	1.2197	1.1927	2.985	1.6662
5	1.59217	1.5763	1.3497	1.3294	3.388	1.8995

The dynamic analysis presented later also confirms the existence of this lowest frequency. It is clear that the theoretical and the BEM methods failed to capture the lowest mode natural frequency. The discrepancy may be attributed to the consideration of shallow water equation in deriving the theoretical results for such tanks. A truncated diverging tank can be divided in two parts. A central rectangular section with width $2h_0 \tan\alpha$ and height $h - h_0$ and two right angled triangular wedges on two sides. Shallow water equation may not be the most suitable assumption for the central portion of the tank with uniform depth of water. Due to the triangular descretisation elements in the present computation, boundary features of the fluid over the inclined boundries as well as of the fluid in the central rectangular section are captured accurately, hence the lowest natural frequency is successfully captured. To verify this fact the natural frequencies for a rectangular tank of dimension 2a=0.4m and h=1m as well as for a diverging tank

with h=1m and $\alpha = \frac{\pi}{4}$ are also computed and persented in Table (3). It is observed that the theoretically computed natural frequency (Ibrahim[2]) is very close to the natural frequency of the diverging tank. The theoretically obtained 2nd mode frequency of the TD tank is also very close to the second mode frequency of the diverging tank. Similarly, the theoretically computed higher mode frequencies are also comparable to the corresponding frequencies of the diverging tank. It shows that the BEM and the theoretical equation is only capable of computing natural frequency of the diverging section where shallow water equation is applicable but not for the combined section of a truncated diverging tank. Natural frequencies of converging (C) tanks as shown in Fig (1d) are also computed with tank parameters h=1.0 m, h_0=0.1m and semi angle $\alpha = \frac{\pi}{4}$ and $\alpha = \frac{\pi}{6}$ and compared with theoretical results from Ibrahim [2] in Table (4). It is observed from the results in Table (4) that the percentage error between the theoretical and the numerical results for a converging tank is higher than that for a diverging (D) tank.

Table (4): Natural frequency of converging tanks h=1.0m, ho=0.1m

	$\alpha = \frac{\pi}{4}$			$\alpha = \frac{\pi}{6}$		
Modes	Ibrahim [2]	Present Numerical	%error	Ibrahim [2]	Present Numerical	%error
1	2.229	2.175	2.39	2.730	2.857	-4.64
2	3.152	2.983	5.36	3.861	4.118	-6.65
3	3.861	3.696	4.26	4.729	5.297	-12.03
4	4.458	4.273	4.15	5.460	6.326	-15.86
5	4.984	4.766	4.38	6.105	7.207	-18.06
6	5.460	5.116	6.31	6.687	7.856	-17.48

The discrepancies may be attributed to the smaller free surface width in a “C” tank. It restricts the divison of the free surface to a smaller number of elements by the proposed triangular descretisation scheme, which limits the accuracy of computation. It is also observed that with decrease in semiangle from $\alpha = \frac{\pi}{4}$ to $\alpha = \frac{\pi}{6}$, the level of inaccuracy increases significantly, since due to the change in semiangle the conditional factor, i.e., maximum to minimum angle ratio, of the mesh elements change**.** Computations are also performed for converging tanks with h=1m, $\alpha = \frac{\pi}{4}$, h_o= 0.2m and 0.3m and the results are presented in Table (5). It is observed that with decrease in height of the tank the numerical results change from an underestimate to an overestimate. This happens due to the fact that with other conditions remaining unchanged the number of triangular elements decreases with the decrease in liquid depth.

Table (5): Natural frequency of converging tanks h=1.0 m, $\alpha = \frac{\pi}{4}$

	ho=0.2m			ho=0.3m		
Modes	Ibrahim [2]	Present Numerical	%error	Ibrahim [2]	Present Numerical	%error
1	1.573	1.534	2.48	1.277	1.224	4.15
2	2.229	2.109	5.38	1.820	2.020	-10.99
3	2.730	2.613	4.29	2.229	2.470	-10.81
4	3.152	3.021	4.16	2.574	2.761	-7.26
5	3.524	3.370	4.37	2.878	3.046	-5.84
6	3.861	3.617	6.32	3.153	3.769	-19.54

3.1.2 Effect of inclined boundaries

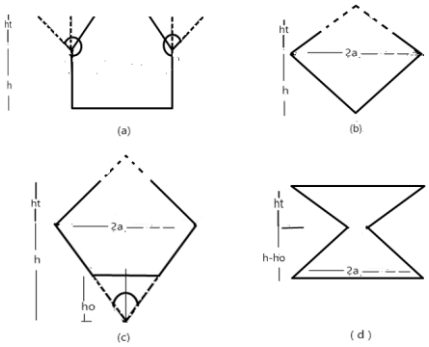


Fig (3) Different composite shaped tanks a) R–C/R–D tank, b) D-C tank, c) Truncated D–C tank, d) C–D tank.

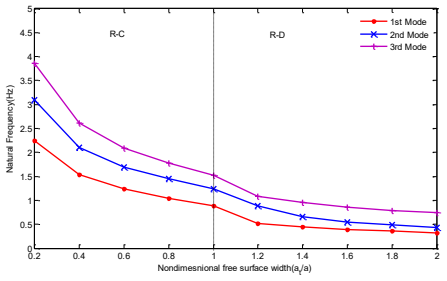


Fig (4) Variation in natural frequency of a R tank due to addition of C or D section (2a=1m, h=1m).

Converging and diverging tanks are generated by changing the inlclination of the sides of the rectangular tanks. The effect of change of inclination of tank side boundary is studied by computing natural frequency of the tank with sides inclined at different angles. A rectangular tank with 2a=2.0m and h=0.9m is considered. Inclination of the side boundary is varied to form convergent (C) and divergent (D) tanks with different semi angles α . Results are also shown in Fig (2). It shows that variation in the two lowest mode natural frequencies due to increase in inclination of the boundary is linear in nature. The natural frequency decreases as inclination of the boundary increases, i.e., as the tank changes from a converging to a diverging shape with different semiangle α. The variation in natural frequency shows a linear relationship with the angle of inclination.

3.1.3 Composite-shaped tanks

Natural frequencies are also computed for various types of composite-shaped tanks formed by combination of elementary 2-D tanks discussed in Sec (3.1.1). Most common section formed due to combination of these shapes are a) Rectangular–Convergent (R-C) b) Rectangular-Divergent (R-D) c) Divergent–Convergent (D-C) d) Convergent–Divergent (C-D) as presented in Fig (3). More complex composite shaped tanks can be designated as D-R-C and C-R-D sections. A very practical tank in most commom use is a prismatic tank which is truncated D-R-C tank. These tanks are designated in a similar manner as above. Natural frequencies are computed by discretising the fluid domain in the tanks by structural triangular finite elements taking due care to capture accurately the boundaries with inernal and external angles. Computed natural frequencies for R-C, R-D, D-C and C-D tanks with dimension are persented in Table (6).

Computations are performed with different grid densities to check the convergence of the computational results. Computational results for tanks of five different shapes are found to converge with different element numbers validating the computational results and the applicability of triangular elements in capturing re-entrant angles accurately.

Table (6): Computational results of natural frequency for different composite tanks

Sl No	Tank shape	Tank parameter(m)	N	Frequency (Hz)		
				1st	2nd	3rd
1	*R-C*	*(2a=2,h=1),h_t=0.5*	500	0.9615	1.3220	1.6319
			2200	0.9553	1.3022	1.5914
2	*R-D*	*(2a=1,h=0.5), h_t=0.5*	250	0.5055	0.7610	0.9946
			1000	0.5052	0.7606	0.9917
3	D-C (Star shaped)	h=1, h_t=0.2, $\alpha = \frac{\pi}{4}$	50	0.6881	1.032	1.2904
			200	0.6816	1.0106	1.2516
4	Truncated (D-C) (Truncated star shaped)	h=1.0,h_0=0.2,h_t=0.2, $\alpha = \frac{\pi}{4}$	92	0.9724	1.5332	1.8818
			368	0.9391	1.4657	1.8743
			1473	0.9289	1.4431	1.8400
5	C-D	*h-ho=0.5, h_t=0.5,* $\alpha = \frac{\pi}{4}$	300	0.5078	0.7613	0.9954
			1200	0.5072	0.7609	0.9924

3.1.4 R-D and R-C tanks

Dynamic behaviour of R-D and R-C tanks is studied to evaluate the effect on natural frequency of a rectangular (R) tank due to the addition of a convergent (C) or Divergent (D) section. Variation in natural frequency is plotted against the nondimensional free surface width of the tanks and persented in Fig (4). It is observed that addition of a convergent section increases the natural frequency of the tank, but the addition of a divergent section reduces the natural frequency of the original tank. In case, the inclination of the extended portion become π/2, the extension just adds to the height of the bottom tank and the natural frequency remains constant with the increase in height of the tank.

3.1.5 D-C and C-D tanks

Effect on the natural frequencies of a divergent (D) and a convergent (C) tank due to the addition of a convergent (C) and divergent (D) section as well as a truncated D-C tank is presented here. Section paramaeters are kept fixed with semi angle $\alpha = \frac{\pi}{4}$ and h=1m for each case. It is observed that when a C tank is added above a D tank, the natural frequency of the composite tank is higher than the original D tank. The opposite happens when a D tank is added above a C tank. Addition of a convergent section leads to increase in frequency and addition of a divergent section leads to decrease in natural frequency of original tanks. The increase in natural frequency in case of D-C tank is considerably larger over the range of a_t/a but in case of C-D tanks the decrease in natural frequency is negligible beyond the nondimensional free surface width a_t/a >2. Truncation in a D-C tank is also found to result in reduction of natural frequency of the original tank.

Polynomials correlating the variation of three lowest mode natural frequencies with the nondimensional free surface width $\tau = \frac{a_t}{a}$ are presented for the three types of tanks in Table (7). It is observed that variation of natural frequency in all these tanks can be closly represented by a cubic polynomial in nondimesnional free surface width "τ". It is also observed that the C-D tank has the flattest curve.

Table (7) Polynomial representing varation of natural frequency for various tanks ($\tau = \frac{a_t}{a}$)

Sl.No.	Tanks	Mode	Polynomial
1	D-C	1st	$f_{\tau 1} = -2.7\tau^3 + 6\tau^2 - 5.1\tau + 2.3$
		2nd	$f_{\tau 2} = -4.7\tau^3 + 10\tau^2 - 7.9\tau + 3.3$
		3rd	$f_{\tau 3} = -5.7\tau^3 + 12\tau^2 - 9.9\tau + 4.1$
2	Truncated D-C	1st	$f_{\tau 1} = -2.1\tau^3 + 4.7\tau^2 - 4.2\tau + 2$
		2nd	$f_{\tau 2} = -3.6\tau^3 + 8\tau^2 - 6.6\tau + 2.9$
		3rd	$f_{\tau 3} = -4.5\tau^3 + 10\tau^2 - 8.3\tau + 3.7$
3	C-D	1st	$f_{\tau 1} = -0.18\tau^3 + 0.97\tau^2 - 1.8\tau + 1.5$
		2nd	$f_{\tau 2} = -0.26\tau^3 + 1.4\tau^2 - 2.6\tau + 2.2$
		3rd	$f_{\tau 3} = -0.25\tau^3 + 1.4\tau^2 - 2.7\tau + 2.5$

3.1.6 D-R-C and C-R-D tanks

These composite shaped tanks are similar to Fig (3b), Fig (3c) and Fig (3d) with a rectangular section in the middle. There are two internal corners on a side boundary in these sections. Structured triangular grid is constructed in such a manner that the corners fit in the grid perfectly. Computed natural frequencies for these tanks with different grid densities are presented in Table (8). It is observed that with a grid of about 400 elements adequate level of convergence is achived for each of these tanks. Variation of natural frequency in a D-R-C tank is computed with different height (h_c) of rectangular section (R) and persented in Fig (5). It is observed that there is hardly any change in natural frequencies of a D-C tank with height of the central rectangular section. This significantly shows that the natural frequency is basically dependent upon the parametric charecterisitcs of the free surface and not on the internal shape of containing volume

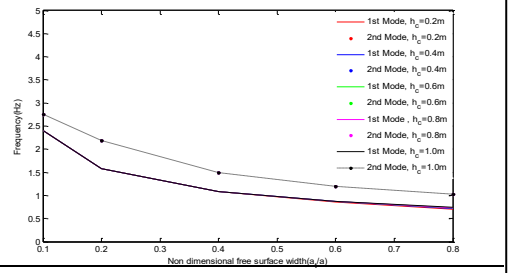


Fig (5) Variation of natural frequency in D-R-C tanks with (a_f/a) for different height

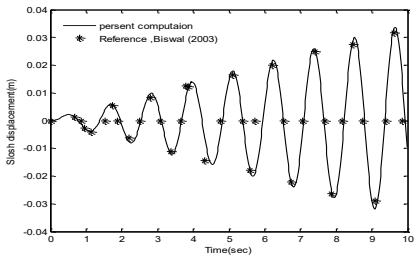


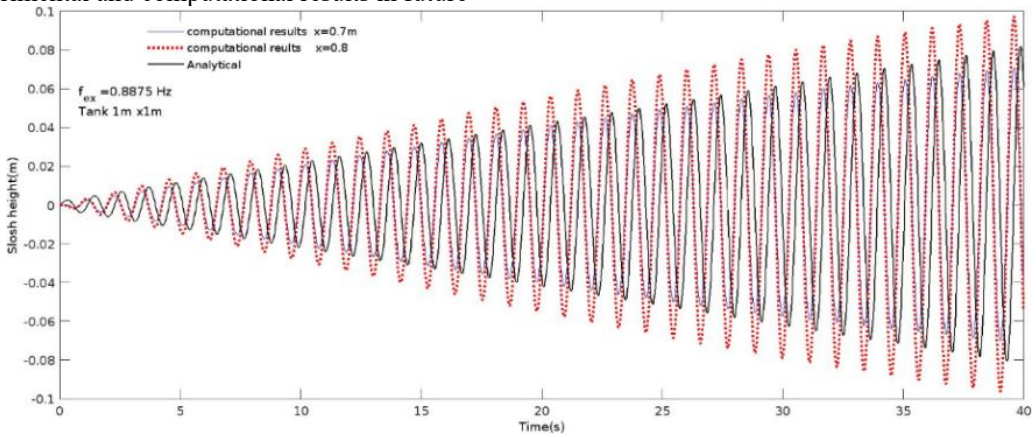
Fig (6a) Displacement at the top left node of rectangular rigid tank (2a=1m, h=1m) at excitation amplitude of 0.001 N/m^2 and frequency 5.55 rad/s.

Table (8) Natural frequency of D-R-C, truncated D-R-C and C-R-D tanks.

Sl No	Tank parameter	N	Frequency (Hz)		
			1st mode	2nd mode	3rd mode
1	h=1m,h_c=0.2m,h_t=0.2m				
	(D-R-C)	118	0.7277	1.0482	1.300
	Star shaped with mid rectangular section	432	0.7211	1.026	1.2602
		1728	0.7192	1.0198	1.2448
2	*h=1m,h_c=0.1m,h_t=0.9m,ho=0.2m*	116	0.6225	0.9199	1.1434
	Truncated D-R-C	464	0.6188	0.9064	1.1160
		1856	0.6177	0.9023	1.1068

3.2 Forced response

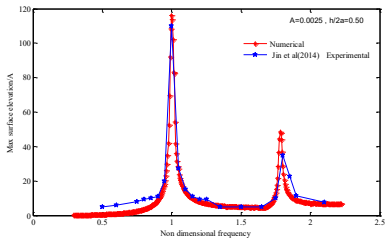
Forced response of a rectangular tank of 2a=1m, h=1m is computed numerically by integrating the nonhomogenous Eq (5) in time at an excitation frequency of 0.88 Hz and amplitude of A=0.001 N/m^2. Computed response over 20s is compared with computed rsults of Biswal et al. [21] and presented in Fig (6a). The results compare well validating the present numerial approach using a triangular discretisation scheme. The simplified analytical tool is also used to calculate the free surface response using Eq. (11) with the natural frequency of a 1 m × 1 m rectangular tank and an exciation frequency of 5.578 rad/s (0.8875Hz). Computed results are presented in Fig (6b) and compared with the computational response at two nodal points on the free surface. Analytical response appears to match in terms of periodicity with a small lead in time but with slightly lower amplitudes. Since the analytical response is obtained with only the fundamental natural frequency, the comparison clearly reveals the marginal influence of the higher modes. This confirms that Eq (11) respersents the free surface response of an undamped fluid system subjected to harmonic oscillation reasonably and can be used for quick comparison of experimental and computational results in future



Fig(6b) Comparision of analytical and numerical computational results.

3.2.1 Frequency response

Numerically computed frequency response of a rectangular tank is plotted by computing the maximum free surface displacement against the excitation frequency and the result is compared with the available results of Jin et al. [25] as shown in Fig (7). The present frequency response curve compares very well with the reference.



Fig(7) Frequency response curve in a rectangular tank h=0.5m, 2a=1m

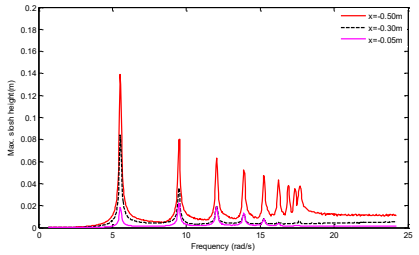


Fig (8) Frequency response curve in a "R" tank at nodal points along free surface

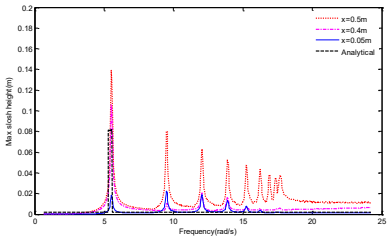
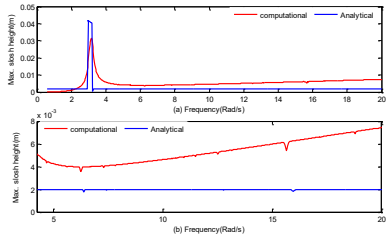


Fig (9) Analytical and numerical frequency response in "R" tank (h=1m, 2a=1m).



Fig(10) Comparision of analytical and computational frequency response in a diverging (D) tank h=1, $\alpha = \frac{\pi}{4}$

Maximum responses at different nodal points along the free surface are also persented in Fig (8). It is observed that the maximum response reduces as the nodal points shift away from the side wall of the tank. The Eq (11) and Eq (12) can also generate similar free surface response where the contributions from the higher modes will be absent. Therfore, the maximum free surface response is also computed using these equations for 40s duration and compared with the computed results for a rectangular (R) and a divergent (D) tank which shows sbustantialy different resonance behaviour at higher modes. The analytical frequency responses are presented in Fig (9) and Fig (10) along with the numerical frequency responses for both the tanks. It is observed that the analytical response correctly predicts the peak near the 1st mode natural frequency in a rectangular tank with the maximum response more than the response at the nodal point -0.05m away from the middle of the free surface.Similarly, in a diverging tank, the analytical frequency response is again found to be similar to the computational response but with a relatively smaller magnitude indicating lower maximum response in a divergent tank as compared to a rectangular tank. There are no discernible peaks corresponding to the higher mode frequencies as persented in Fig (10 b).

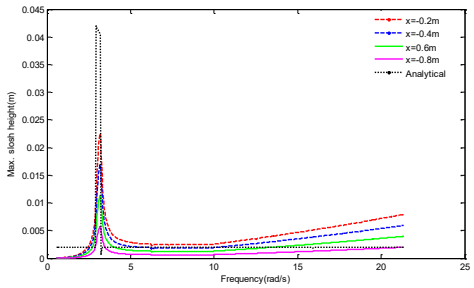
Table (9) Relative slosh alleviation in different types of tanks

Sl No.	Tanks	Dimension		$\gamma_r = \frac{\eta_{A\max}}{\eta_{C\max}}$
		Original	Composite	
1	R	h=1m, 2a=1m	Nil	0.5882
2	D	h=1m, $\alpha = \frac{\pi}{4}$	Nil	1.3400
3	C	2a=2m, h=0.5m	Nil	0.6956
4	R-D	h=0.5m, 2a=1m	2a=1m, ho=0.5m	0.7575
5	R-C	h=1m, 2a=2m	2a=2m, ho=0.5	0.9319
6	D-C	h=1m, $\alpha = \frac{\pi}{4}$	h=1m, ho=0.5m	0.7752
7	C-D	2a=2m, h=0.5m	ho=0.5m	1.9829

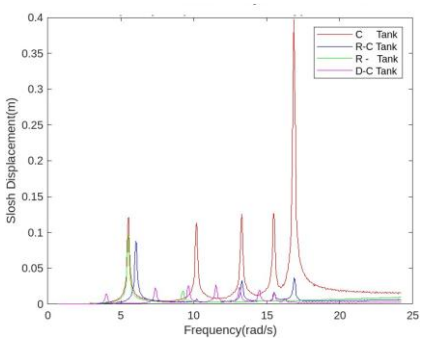
It is observed from Fig (11) that the response curves are similar in nature near the resonance but the response at the other nodal points is less than that computed at the left most node. Changes in response at higher modal frequencies are also noticeable but the magnitudes are very small. Higher mode frequency responses can be generated by superposing the modal frequency in Eq. (11) and Eq. (12). It is observed that in case of a divergent tank the maximum response at the wall node is much smaller than the

elementary analytical frequency response and responses at the other nodal points are still smaller. The relative effect on slosh response due to geometrical changes like R-D, R-C tanks as well as C-D and D-C tanks is presented in Table (9).
It is observed that the C-D tanks have the highest relative slosh suppression when compared with rectangular tanks. This indicates that the geometricaly modified tanks or composite shaped tanks, in general, leads to improvement in slosh alleviation properties. R-C and R-D tanks have better slosh alleviation properties than a R tank. Similarly, the slosh control property is found to improve in C-D tank than that in a C tank

The only exception is a divergent tank where the modification leads to an aggravation in the sloshing characteristic of the tank. Therfore, geometrical modification can be applied successfully as an alternative to control sloshing properties of various types of tank in lieu of active methods like provision of submerged or surface piercing baffles or screens which may lead to complex fluid flow charecterisitcs within the fluid domain.



Fig(11) Comparision of analytical and computational frequency response at different nodal points on the free surface in a divergent tank



Fig(12) Comparision of frequency response of different composite shaped tanks

To further understand the damping bahviour of composite shaped tanks the maximum free surface displacement of various composite shaped tanks with equal free surface width of 1 m have been studied for a range of excitation frequencies. The computed results are presented in Fig (12). It is observed that for an equal free surface the maximum displacement is obtained in a convergent tank. However, the largest displacement does not occur at close to the lowest modal frequency but at a higher modal frequency indicating slosh suppression near the lowest mode of frequency. A D-C tank is found to have the least response in all modes. An R-C tank response is found to be less them a rectangular tank. Therfore, the geometrical modifications of a container is found to be beneficial in terms of slosh suppression properties.

4. CONCLUSION

. Triangular elements allow accurate and refined finite element modelling around inclined boundaries and reentrant angles avoiding singularities in a very simplified manner. Finite element pressure formulation can be used for natural and forced dynamic analysis of 2D elementary tanks as well as their geometrical derivatives more effectively than BEM and analytical/theoretical methods.

Theoretical equations as well as BEM proposed for truncated divergent tanks fail to capture the lowest mode frequency since in all practical cases the shallow water wave theory is not applicable, at least, over a part of the tank volume. Natural frequency is mostly dependent upon the free surface width and the boundary conditions at the edge of the tank. Increase in inclination of the side boundary of a tank results in a linear reduction in natural frequency.

Addition of a divergent section causes a decrease in natural frequency, while the addition of a convergent section causes an increase in natural frequency. Variation in the three lowest mode frequencies can be approximated by cubic polynominals in nondimensional free surface width.

Analytical solutions for a damped and undamped equivalent mass spring model can approximately represent the elementary response for various types of tanks under similar excitation forces. The approximate analytical solution can be used for a quick comparison of the effectiveness of slosh control measure in tank systems. Divergent tanks, amongst the elementary tanks, produce the maximum slosh suppression and amongst the composite shaped tanks the C-D tanks provide the highest slosh suppression.

ACKNOWLEDGEMENTS: I express sincere thanks to A&N Adminstration for sponsering my Doctoral study program at IIT Kharagpur

REFERENCES

1. Abramson H.N., (1966), Dynamic behavior of liquid in moving container, NASA SP 106.
2. Ibrahim R. (2005), Liquid sloshing dynamics, Cambridge University Press, Cambridge 2005
3. Faltinsen O.M. and Timokha A. (2009), Sloshing, Cambridge University Press, Cambridge 2009
4. Faltinsen O.M. and Timokha A. (2014), analytically approximate natural sloshing modes and frequencies in two-dimensional tanks, European Journal of Mechanics-B Fluids, Vol 47 (2014) 176-187

5. Pal N.C. (1999) , Coupled slosh dynamics of liquid filled laminated composite containers, Ph.D Thesis, Indian Institute of Technology, Kharagpur, India, pp. 80, 1999.
6 Biswal K.C (2003), Dynamic response of liquid filled composite containers with baffle considering liquid structure interaction, Ph.D. Thesis, Indian Institute of Technology Kharagpur.
7. Ghaemmaghami A.R. Kianoush M.R.(2010), Effect of wall flexibility on dynamic response of concrete rectangular liquid storage tanks under horizontal and vertical ground motion, Journal of structural engineering @ASCE Vol 136(4),Pp441-451
8. Nayak S.K and Biswal K.C.(2013)Quantification of seismic responce of partialy filled rectangular liquid tank with submerged block, Journal of earthquake engineering vol 17:7 Pp 1023-1062
9. Nayak S.K. and Biswal K.C.(2015),Fluid damping in rectangular tank fitted with various internal objects- An experimental investigation Ocean engineering Vol 108, Pp 552-562
10. Mitra S., Upadhyay P. P. and Sinhamahapatra K.P. (2008), Slosh dynamics of inviscid fluids in two dimensional tank of various geometry using finite element method, International Journal of Numerical Method in Fluids, 2008,56: 1625-1651
11. Laha M.K. and Dutta S.(2000) Analysis of the small amplitude sloshing of a liquid in a rigid container of arbitrary shape using a low oder boundry element method , International Jounral of Numericaal Methods in Engineering, vol 47, Pp 1633-1648
12. Jaiswal O.R, Kulkarni Shardha and Pathak Pavan (2009), The 14th world conference on earthquake engineering, October 12-17, Beijing, China
-13. Zhang C. (2015a), Analysis of liquid sloshing in LNG carrier with Wedged shaped tank, Ocean Engineering, Vol 105, 304-317
14. Zhang C. (2015b), Application of improved Semi-Lagrangian procedure to fully non linear simulation of sloshing in non wall sided tanks, Applied Ocean Research, Vol 51, 74-92.
-15. Zhang C. (2016), Non Linear simulation of resonant sloshing in wedged tanks using Boundry Element Method, Engineering Analysis with Boundry Elements, Vol 69, 1-20
16. Kumar A. and Sinhamahapatra K.P(2021), Dynamic analysis of fluid in 2 –D prismatic tanks with triangular finite element, International journal of fluid mechanics and research vol 48,Issue -1, pp 37-61
17. Agresta Antonio, Cavalagli Nicola, Biscarini Chaira, Ubertina Flipa (2021), Effect of bottom geometry on the natural sloshing motion of water inside tanks: an experimental analysis, Journal of Applied mechanics Vol -11, 609
18. Bole Ma,Baozeng Yue, Feng Liu,Yu lu Cuicui Liu,Xiaodong Guan(2023), Study on the dynamical charecteristics and experimental validation for liquid sloshing in a common Bulkhead Tank, Acta Mechanica sinica vol 39, Article no 523127
19. Kumar A and Sinhamahpatra K.P (2013), Effect of porous internal component on liquid slosh dynamics,Journal of porous media Vol 16(8), pp, 2013.
20. Kumar A and Sinhamahapatra K.P.(2014) Dynamics of partially filled rectangular rigid tank with surface piercing element , international journal of fluid mechanics and research ,2014
21. Biswal K.C., Bhattacharya S.K., Sinha P.K (2003), Free vibration analysis of liquid filled tank with baffle, Journal of Sound and Vibration, vol. 259(1), 177-192, 200
22. Kreyszig Erwin, Advance Engineering Mathematics (Fifth Ed) Wiley Eastern Limited Pg No 103-112
23. Aguilar Arevalo L.M., Robledo-Sanchez C., Arroyo Carrasco M.L. and Mendez Otero M.M.(2012), The principle of superposition for waves : The amplitude and phase modulation , Applied mathematics &information science vol6:2 Pp 307-315
24. Zhe Hu X Zhang Xiaown Li , Yan Li (2018), On Natural frequencies of liquid sloshing in 2 –D tanks using Boundry Element Method, Ocean engineering,153, pp 88-103
25. Heng Jin, Yong Li, Hua-Jin Li (2014), Experimental study on sloshing in a tank with an inner horizontal perforated plate, Ocean engineering 82, pp 75-84