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Mode shapes for free vibration of clamped plates with various geometries based on an quadrilateral element

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ABSTRACT

This work aims at presenting mode shapes for free vibration of clamped plates with various geometries based on a four-node quadrilateral element, CP-DSG4, related to discrete shear gap (DSG) strategy and adding a center point (CP). The efficiency of the CP-DSG4 element is demonstrated through some numerical tests.

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1. INTRODUCTION

The Reissner–Mindlin theory, known as the first-order shear deformation theory (FSDT), is popular and has been intensively used for investigating plate structures. In terms of numerical simulation, the theory only requires the shape functions to satisfy C0-continuity, which is much more convenient than others. Reissner–Mindlin theory is applicable for both moderately thick and thin plates; however, it suffers from the so-called shear locking problem in the sense of thin plate analysis. A simple remedy, namely selective reduced integration in [1], may be used, i.e., fewer integration points than usual are used for computation of the shear stiffness. Besides, a large number of techniques have been proposed so far to treat the shear locking in the context of the finite element method (FEM); for example, the family of MITC elements based on mixed formulation [2–4], the assumed strain [5–8], etc. The technique of discrete shear gap, originally proposed by Bletzinger et al. [9], can also be considered as assumed strain, in which the shear gap is approximated from the nodal displacements, i.e., deflection and rotations. The formulation is quite straightforward, and recently, Nguyen et al. [10] proposed the introduction of a fictitious point located at the center of the quadrilateral element, namely CP-DSG4, to improve the accuracy.

Besides, the analysis of free vibration of the plate is an indispensable part related to the actual behavior of this type of structure. There are many studies related to this topic, only some of which are mentioned here [11-21]. Clearly, [11] reported a modeling and experimental study on the free vibration characteristics of plates with curved edges. The model was established by the first-order shear deformation theory and Chebyshev differential quadrature method. Based on the Kirchhoff plate theory, the stiffness and mass matrices were calculated using the finite element method for determining natural frequencies as in [12]. In [14], the authors presented the modal analysis of a thin rectangular plate simply supported. The results obtained by the finite element method through Ansys 15.0 are given. Moreover, the [15] represented a finite element analysis of free vibration of isotropic plates with different cutout shapes, areas, locations, and aspect ratios. Modal analysis was carried out using the Ansys Apdl software to evaluate the fundamental frequencies as well as mode shapes. The free vibration analysis of isotropic thick rectangular plates, based on higher-order shear deformation theory, was given in [16]. The plate theory ensured a zero shear-stress condition at the top and bottom surfaces of the plate and did not require a shear correction factor. The model required inter-element C1 continuity for the transverse displacement. To overcome this hindrance, a new hierarchical p-element with six degrees of freedom per node was developed and used to find natural frequencies of thick plates and so on.

In this article, the CP-DSG4 element is used to achieve mode shapes for free vibration of clamped plates with various geometries.

2. FORMULATION

According to the Reissner–Mindlin plate theory, the displacement fields at an arbitrary point can be given by

$$u(x,y,z) = -z\theta_x(x,y) \tag{1}$$

$$v(x,y,z) = -z\theta_y(x,y) \tag{2}$$

$$w(x,y,z) = w_o(x,y) \tag{3}$$

in which w_o is the vertical displacement at the mid-surface and θ_x, θ_x are the rotations about y- and x-axes. The strain components are then obtained by

$$\boldsymbol{\varepsilon}_b = \begin{bmatrix} -z\theta_{x,x} \\ -z\theta_{y,y} \\ -z\left(\theta_{x,y} + \theta_{y,x}\right) \end{bmatrix} \tag{4}$$

$$\boldsymbol{\varepsilon}_s = \begin{bmatrix} w_{o,x} - \theta_x \\ w_{o,y} - \theta_y \end{bmatrix} \tag{5}$$

For homogeneous and isotropic materials, the material matrices $\mathbf{D}_b$ and $\mathbf{D}_s$ characteristic for bending and shearing are given by

$$\mathbf{D}_b = \frac{Eh^3}{12\left(1-\nu^2\right)} \times \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{6}$$

$$\mathbf{D}_s = \frac{5}{6} \times \frac{Eh}{2(1+\nu)} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$

with discrete shear gap (DSG) strategy and adding a center point (CP) as in [10] and Figure 1, the $\mathbf{D}_s$ can be replaced

$$\mathbf{D}_s = \frac{5}{6} \times \frac{E}{2(1+\nu)} \times \frac{h^3}{h^2 + 0.1 l_{el}^2} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{8}$$

where l_{el} is the longest side of the element and h is thickness of the plate.

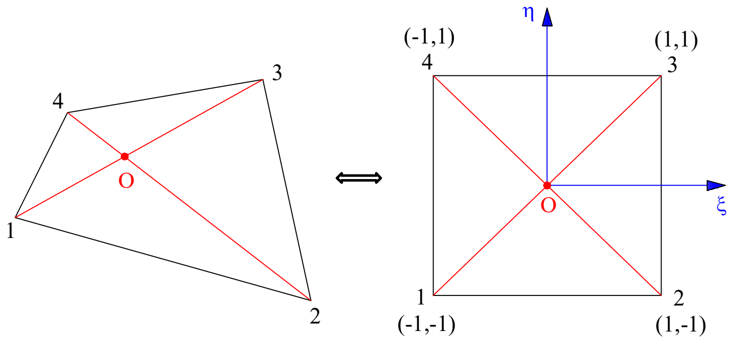

Figure 1. CP-DSG4 element

3. RESULTS AND DISCUSSIONS

In this section, three types of clamped plates will be considered, as shown in Figure 2 and Table 1.

Firstly, to verify the reliability of the applied element, a rectangular clamped plate is considered with the parameters shown in Table 2. The obtained results converge quite well when compared with the others given in the literature [21] based on the Rayleigh-Ritz method.

Then, the free vibration frequency results associated with the first eight mode shapes are shown in Table 3 and Figures 3 to 5 for all three types of plates, respectively.

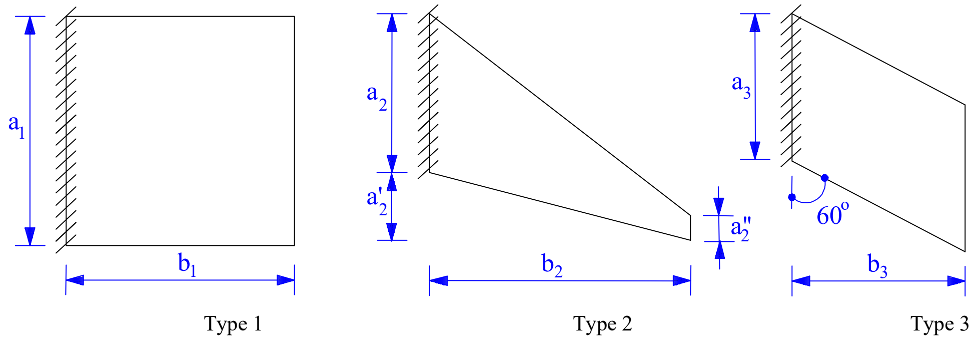

Figure 2. Three types of clamped plate

Table 1. The properties of plates

	$E = 70\times10^9$ Pa, $\nu = 0.3$, $\rho = 1700$ kg/m^3
Type 1	$a_1/b_1 = 1$, $a_1/t = 10$
Type 2	$a_2 = 2$, $a'_2 = 0.7$, $a''_2 = 0.2$, $b_2 = 2.5$, $a_2/t = 10$
Type 3	$a_3/b_3 = 1$, $a_3/t = 10$

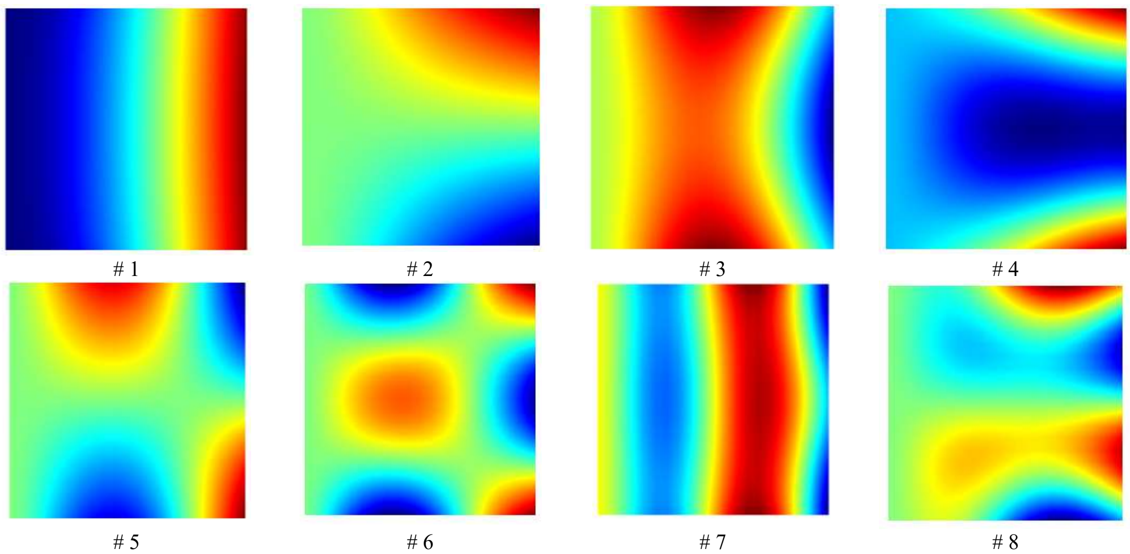

Figure 3. The first eight mode shapes of the Type 1 plate

Table 2. The comparison of natural frequencies of the Type 1 plate

Comparison (Hz)	$E = 72\times10^9$ Pa, $\nu = 0.3$, $\rho = 2840$ kg/m^3, $a_1 = 0.151$, $b_1 = 0.275$, $t = 0.000381$			
Type 1	f_1	f_2	f_3	f_4
[21]	4.2536	16.6120	26.6572	55.1597
Article	4.2099	16.7219	26.3693	55.2236

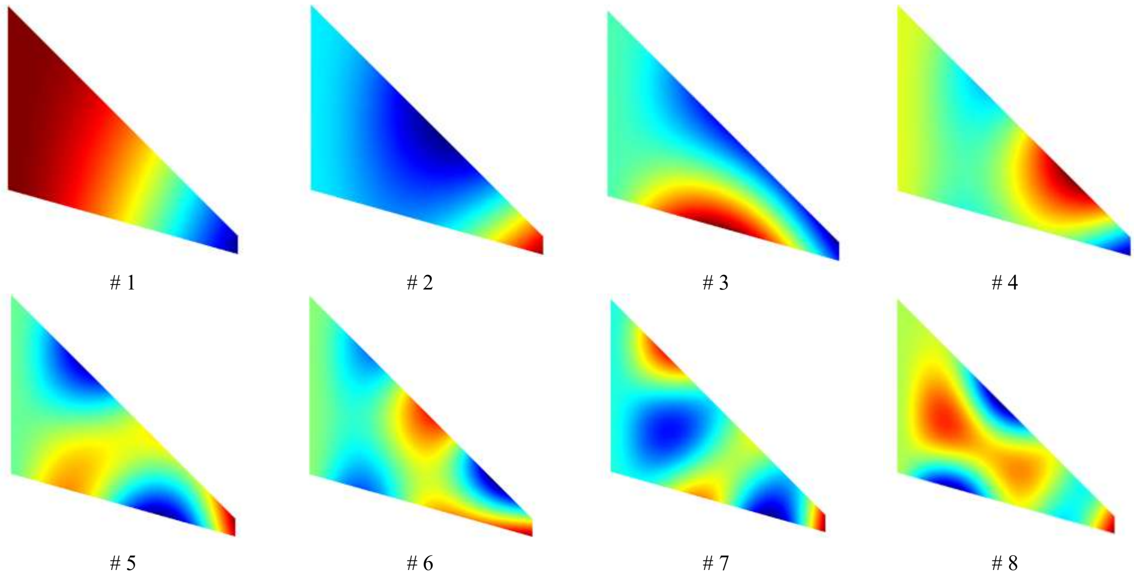

Figure 4. The first eight mode shapes of the Type 2 plate

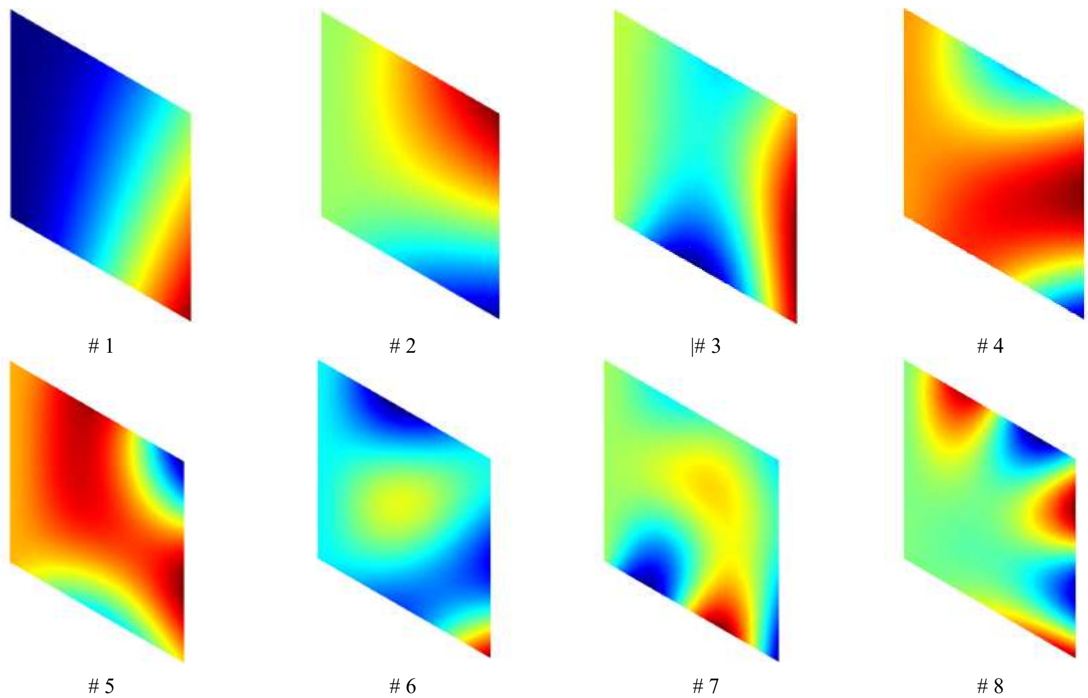

Figure 5. The first eight mode shapes of the Type 3 plate

Table 3. The natural frequencies for three types of plate

(Hz)	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
Type 1	106.0960	250.0298	624.5476	792.1903	879.3625	1481.9335	1698.4709	1785.4485
Type 2	46.4596	190.1685	280.0774	460.6531	632.3063	830.631	995.3096	1224.1452
Type 3	1.2147	2.9119	7.8633	8.0516	12.8582	15.78072	22.7804	23.0981

4. CONCLUSION

This article briefly shows the mode shapes for free vibration of three specific types of clamped plates based on the quadrilateral element named CP-DSG4. With a mesh of 20x20 elements, the results are quite clear and have the necessary reliability. From here we can use the CP-DSG4 element to continue analyzing other mechanical behaviors for plate structures and can extend the problem to different kinds of materials.

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