International Journal of Engineering and Applied Physics (IJEAP)
Vol. 4, No. 2, May 2024, pp. 989~993
ISSN: 2737-8071

Transverse deflection of sandwich beam with 2D-FG skins and ceramic core

Lan Hoang That Ton*
* Dept. of Civil Engineering at HCMC University of Architecture, Vietnam.

Article Info

Article history:

Received April 16, 2024
Revised April 30, 2024
Accepted May 15, 2024

Keywords:

Sandwich beam
2D-FG skin
Ceramic core
Transverse deflection

ABSTRACT

The transverse deflection of a simply supported/clamped (SS/CC) sandwich beam with two directional functionally graded (2D-FG) skins and a ceramic core, namely SW2DC, is given under uniform load. The finite element analysis based on Matlab software is applied to survey the influences of material properties on deflections. The results are then shown and compared with other results in the references to end this study.

Corresponding Author:

Lan Hoang That Ton,
Department of Civil Engineering,
HCMC University of Architecture, Vietnam.
Email: tltechonlinesom@gmail.com

1. INTRODUCTION

Functionally graded material has become one of the smart materials, and it is widely used in many different fields. For example, lots of products related to the defense industry, like nuclear tanks, spacecraft, etc., are produced based on the above material. Due to the high applicability of functionally graded material, many studies related to various theories have been given to comment on the mechanical behavior of functionally graded structures [1–5]. There are three types of structures: beam, plate, and shell, but researchers are usually interested in beam structures because of their wide applications. Using a simple beam model [6–8] helps us reduce the computational cost while keeping the resulting error within the allowable range. On the other hand, the few published papers on the bending behavior of functionally graded sandwich beams can be listed here [9–15]. The main objective of the work in [9] was to propose a coupling of the radial point interpolation method with variable shape parameters to the high-order continuation method for analyzing the geometrically non-linear behavior of porous functionally graded isotropic and sandwich Timoshenko beams by taking into account the non-linear shear deformation. The functionally graded and functionally graded sandwich beams effective material properties were assumed to vary continuously in the thickness direction according to a power-law index. The porosity was modeled as the stiffness reduction criteria and included in the mixture rule. The strong form of the non-linear equations governing porous functionally graded and functionally graded sandwich beams was obtained using the principle of virtual displacements in the framework of first-order shear deformation theory. The high order continuation method was used to compute the solution of the non-linear problem by a transformation to recursive succession of linear problems using Taylor series expansion. The continuation technique was used to obtain the complete solution path in a step by step manner. In the investigation of [10], linear and nonlinear bending analyses of sandwich beams with functionally graded cores were determined under different types of distributed loads. These sandwich beams were composed of two isotropic faces and a porous core with different gradients of internal pores. The governing formulation used to describe the beam's linear and nonlinear behavior was constructed from Reddy's third-order shear deformation theory and nonlinear strain–displacement relations of von Kármán. The Gram-Schmidt orthogonalization procedure was adopted to generate numerically stable functions for the displacement field to solve the beam problems with various boundary conditions. Then, the Ritz method was utilized to find out linear and nonlinear bending results in conjunction with the iterative technique. As a first endeavor, the out-of-plane free vibrational behaviors and moving load responses of sandwich curved beams with graphene platelets reinforced composite face sheets and porous core were carried out in [11]. The governing equations were derived based on the first-order shear deformation theory using Hamilton's principle, which were discretized by employing the differential quadrature method and Newmark's method in the spatial and time domains, respectively. To simulate the moving load using the differential quadrature method, the Heaviside function approach was utilized. The effective elastic properties of face sheets were estimated using the Halpin-Tsai micromechanical model. The approach was validated by presenting convergence studies and accuracy verification, followed by parametric studies. The numerical results indicated that by adding a small amount of graphene platelets to the face sheets and core layer, the fundamental natural frequency and the displacement amplitudes under moving loads significantly increased and decreased, respectively, regardless of the beam boundary conditions. The paper [12] was devoted to a sandwich beam with an asymmetric structure under a uniformly distributed load. The first end of the beam was simply supported, while second end was clamped. The individual nonlinear deformation function of a plane beam cross section—the shear effect—with consideration of the classical shear stress formula was analytically developed. Displacements, strains and stresses taking into account the individual nonlinear deformation function for successive layers of this beam were elaborated in detail. The system of two differential equation of equilibrium of this beam, based on the principle of stationary potential energy, was obtained. This system of equations was precisely solved analytically, according to the theory of differential equations. In the paper [13], a novel refined shear deformation beam theory was proposed and applied, for the first time, to investigate the bending behavior of functionally graded sandwich curved beam. The theory was exploited to satisfy the parabolic variation of shear stress distribution along the thickness direction, thereby obviating the use of any shear correction factors. The material properties of functionally graded sandwich beams changed continuously from one surface to another according to a power-law function. Three common configurations of FG beams were considered for the study, namely: single-layer functionally graded beams; sandwich beams with functionally graded face sheets and homogeneous cores; and sandwich beams with homogeneous face sheets and functionally graded cores. The governing equations derived herein were solved by employing the finite element method using a two-noded beam element. The authors in [14] investigated sandwich composite beams using a direct approach that models slender bodies as deformable curves endowed with a certain microstructure. They derived general formulas for the effective stiffness coefficients of composite elastic beams made of several non-homogeneous materials. Special attention was given to sandwich beams with foam cores, which were made of functionally graded or piecewise homogeneous materials. In the case of small deformations, the theoretical predictions were compared with experimental measurements for the three-point bending of sandwich beams, showing a very good agreement. For functionally graded sandwich columns, they obtained the analytical solutions to bending, torsion, and extension problems and compared them with numerical results computed by the finite element method. The paper [15] presented the static behaviour of two-directional functionally graded sandwich beams subjected to various sets of boundary conditions by using a quasi-3D shear deformation theory and the Symmetric Smoothed Particle Hydrodynamics method. The Symmetric Smoothed Particle Hydrodynamics code, which was developed based on the present formulation of the functionally graded sandwich beam, was validated by solving a simply supported conventional functionally graded beam problem and so on.

Returning to this article, the numerical results are easily obtained, and the small error when compared with the results of other documents is a necessary demonstration. This article is constructed in four sections, including the introduction of SW beam, the formulation for SW beam with 2D-FG skins and ceramic core, and some numerical results with a few comments.

2. SANDWICH BEAM WITH 2D-FG SKINS AND CERAMIC CORE

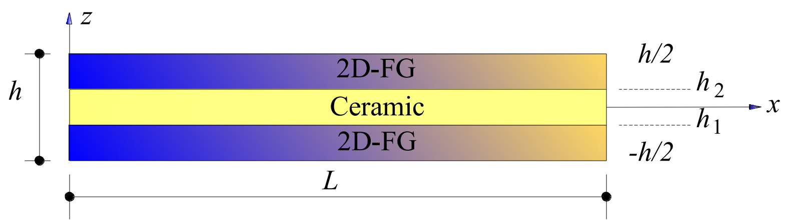

Figure 1. Sandwich beam with 2D-FG skins and ceramic core

$$\begin{cases} P(z) = P_c + (P_m - P_c)\left(\dfrac{-z+h_1}{h/2+h_1}\right)^{n_z}\left(1-\dfrac{x}{2L}\right)^{n_x}, & -h/2 \le z \le h_1 \\ P(z) = P_c, \quad h_1 < z < h_2 \\ P(z) = P_c + (P_m - P_c)\left(\dfrac{z-h_2}{h/2-h_2}\right)^{n_z}\left(1-\dfrac{x}{2L}\right)^{n_x}, & h_2 \le z \le h/2 \end{cases} \qquad (*)$$

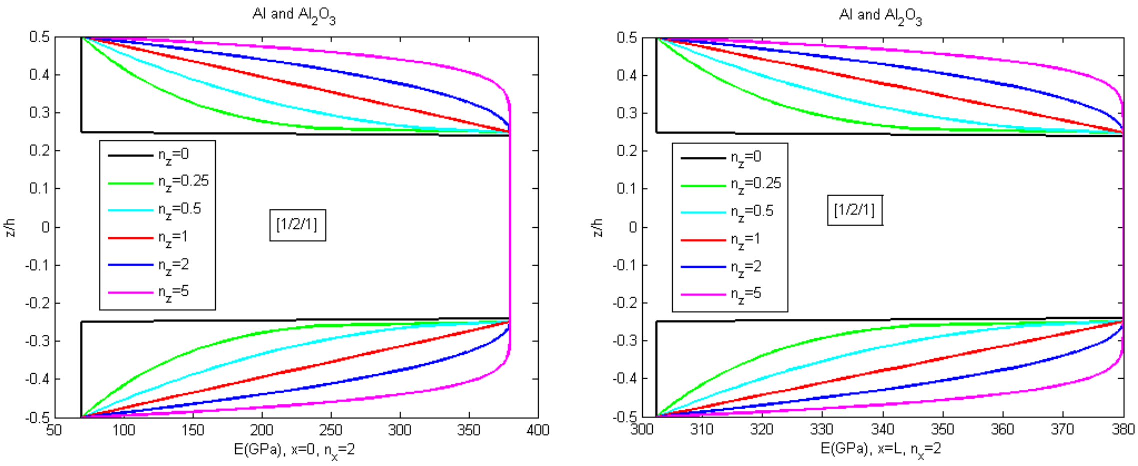

Figure 2. The modification of E with n_x = 2 and ratio of the thicknesses [*1/2/1*]

A SW2DC beam of length L and thickness h is studied. This beam consists of two directional functionally graded skins and ceramic core as shown in Figure 1. The values of material properties $P(z)$ are given in Equation (*). A group of three numbers like "t_1 / t_2 / t_3" is used to denote the ratio of the thicknesses of the bottom-core-top layers. It means the thickness of the bottom layer from $-h/2$ to h_1 is $h.t_1/(t_1 + t_2 + t_3)$, that of the core layer from h_1 to h_2 is $h.t_2/(t_1 + t_2 + t_3)$ and that of the top layer from h_2 to $h/2$ is $h.t_3/(t_1 + t_2 + t_3)$. The effective Young's modulus is demonstrated in Figures 2 with beam containing *Al* and Al_2O_3 materials. On the other hand, based on traditional finite element analysis with a total four degrees of freedom on an element, numerical results can be easily obtained. Two kinds of boundary conditions can be taken as (*SS*) and (*CC*) for the practice of SW2DC beams.

3. NUMERICAL RESULTS

Firstly, the verification of this model is presented for (*SS*) SW2DC beam under a uniform load q. The material properties of beam is provided as metal *Al* (E_m = 70 GPa, ν = 0.3) and ceramic Al_2O_3 (E_c = 380 GPa, ν = 0.3). The maximum transverse deflection is normalized by $\bar{w} = 100E_m h^3 w(L/2)/q/L^4$. The values of normalized deflection for SW2DC beams with L/h = 5 & 20 are given in Table 1 and compared with other results from another beam theory in [15]. It can be seen that the results obtained from this article are approximate with other results.

Table 1. The comparison of the normalized deflections of (*SS*) SW2DC beams

n_x	n_z	*Method*	L/h = 5		L/h = 20	
			[1/1/1]	[1/2/1]	[1/1/1]	[1/2/1]
0	0	[15]	*13.7597*	*10.3450*	*13.4506*	*10.0800*
		Article	13.7144	10.3414	13.2307	9.9656
	0.1	[15]	*11.5719*	*8.9629*	*11.2682*	*8.6934*
		Article	11.5668	8.9224	11.1234	8.5642
	0.5	[15]	*7.8402*	*6.4901*	*7.5572*	*6.2234*
		Article	7.8544	6.4946	7.4921	6.1746
	1	[15]	*6.2095*	*5.3611*	*5.9373*	*5.0983*
		Article	6.2170	5.3623	5.8948	5.0652
	2	[15]	*4.9737*	*4.4839*	*4.7119*	*4.2269*
		Article	4.9737	4.4797	4.6830	4.2025
1	0	[15]	*7.5490*	*6.6017*	*7.2296*	*6.3437*

	Article	7.5585	6.6003	7.1485	6.2583
0.1	[15]	*6.9827*	*6.1379*	*6.6753*	*5.8568*
	Article	6.9866	6.1174	6.6030	5.7890
0.5	[15]	*5.7178*	*5.1238*	*5.4388*	*4.8611*
	Article	5.7110	5.1105	5.3821	4.8106
1	[15]	*4.9904*	*4.5528*	*4.7252*	*4.2959*
	Article	4.9787	4.5374	4.6776	4.2541
2	[15]	*4.3387*	*4.0486*	*4.0816*	*3.7958*
	Article	4.3213	4.0293	4.0428	3.7606

The next problem will be developed by investigating the variation of deflection when changing parameters n_x, n_z, L/h ratio, ratio of the thicknesses for (*SS*) SW2DC beams. The obtained numerical results are depicted in Figures 3 & 4. It is easy to see that when fixing the n_x value and increasing the n_z value, the value of transverse deflection gradually decreases for all cases.

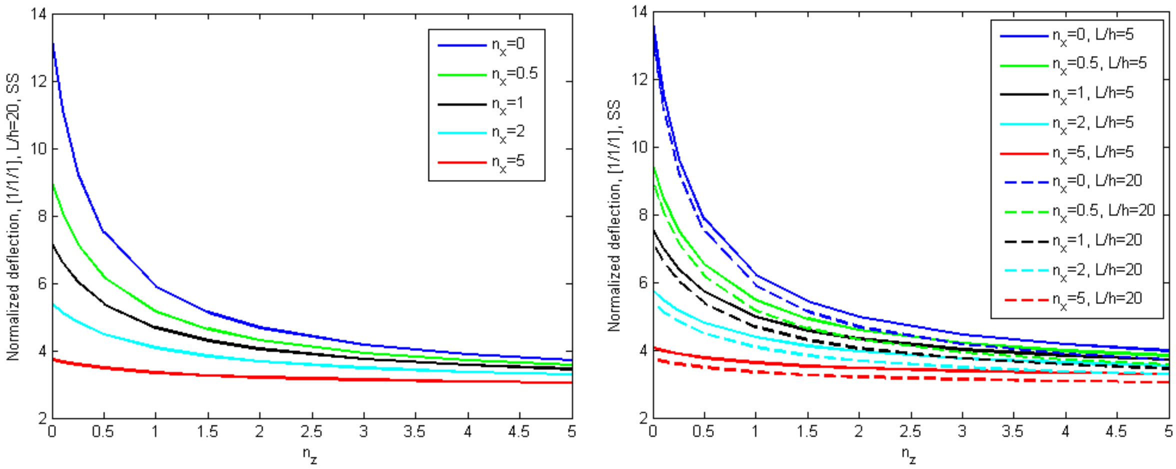

Figure 3. The normalized deflections with *L/h* = 5 & 20, [1/1/1], (*SS*) condition.

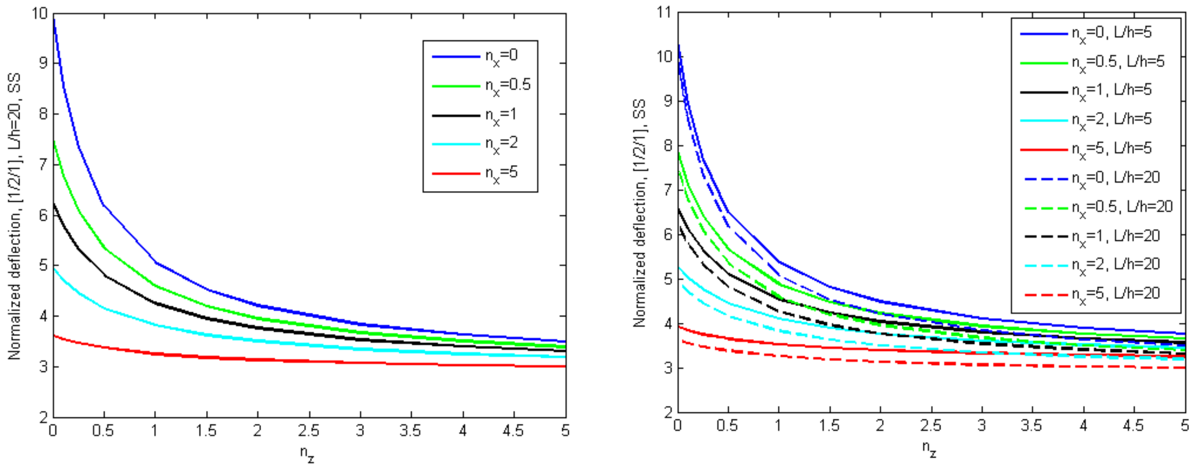

Figure 4. The normalized deflections with *L/h* = 5 & 20, [1/2/1], (*SS*) condition.

Finally, by changing different boundary condition from (*SS*) to (*CC*), numerical results are illustrated quite quickly as shown in Figure 5, respectively.

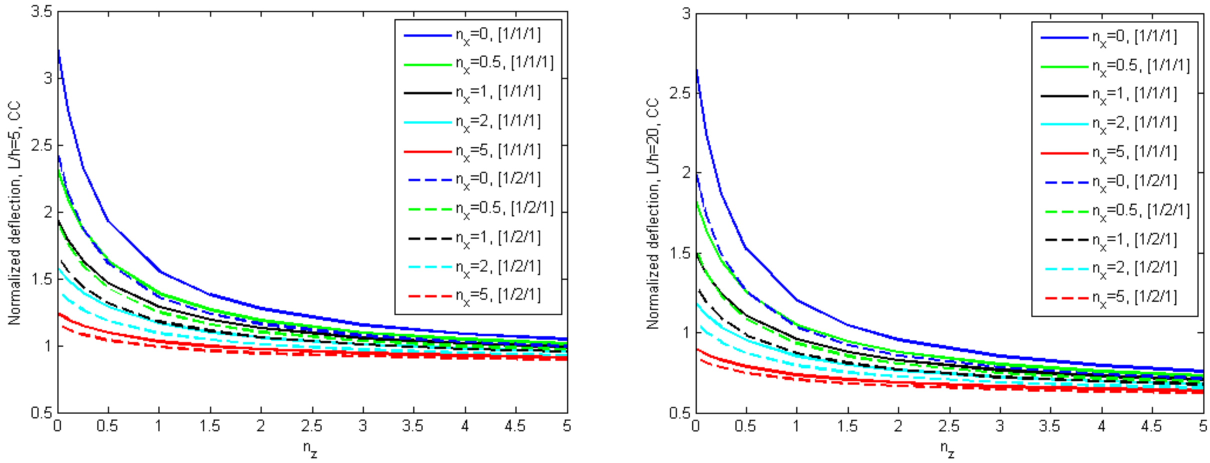

Figure 5. The normalized deflections with *L/h* = 5 & 20, [1/1/1] & [1/2/1], (*CC*) condition.

4. CONCLUSION

In this article, the normalized transverse deflections of SW2DC beams under two different types of boundary condition (*SS*) & (*CC*) are given. The proposed results are approximate with others in references. The main aim is to affirm this easily application to analyze the SW2DC beams with acceptable results.

REFERENCES

[1] A. Pasha and B. M. Rajaprakash, "Fabrication and mechanical properties of functionally graded materials: A review", *Materials Today: Proceedings*, 52, 379-387, 2022.

[2] E. J. Kim, C. M. Lee and D. H. Kim, "Study on the characteristics of functionally graded materials from Ni-20cr to Ti-6Al-4V via directed energy deposition", *Journal of Alloys and Compounds*, 955, 170263, 2023.

[3] D. D. Singh, S. Arjula and A. R. Reddy, "Functionally graded materials manufactured by direct energy deposition: A review", *Materials Today: Proceedings*, 47, 2450-2456, 2021.

[4] Y. Xiong, Z. Han, J. Qin, L. Dong, H. Zhang, Y. Wang, H. Chen and X. Li, "Effects of porosity gradient pattern on mechanical performance of additive manufactured Ti-6Al-4V functionally graded porous structure", *Materials & Design*, 208, 109911, 2021.

[5] D. Khrapov, M. Kozadayeva, A. Koptyug, T. Mishurova, D. Meinel, R. Surmenev and M. Surmeneva, "Geometrical features and mechanical properties of the sheet-based gyroid scaffolds with functionally graded porosity manufactured by electron beam melting", *Materials Today Communications*, 35, 106410, 2023.

[6] H. Qin, Y. Yan, H. Liu, J. Liu, Y. W. Zhang and Y. Liu, "Modified Timoshenko beam model for bending behaviors of layered materials and structures", *Extreme Mechanics Letters*, 39, 100799, 2020.

[7] E. Ruocco and J.N. Reddy, "Analytical solutions of Reddy, Timoshenko and Bernoulli beam models: A comparative analysis", *European Journal of Mechanics - A/Solids*, 99, 104953, 2023.

[8] Y. Kaya and J. Dowling, "Application of Timoshenko beam theory to the estimation of structural response", *Engineering Structures*, 123, 71-76, 2016.

[9] K. Magnucki and E. M. Blandzi, "A refined shear deformation theory of an asymmetric sandwich beam with porous core: Linear bending problem", *Applied Mathematical Modelling*, 124, 624-638, 2023.

[10] S. Mesmoudi, Y. Hilali, M. Rammane, O. Askour and O. Bourihane, "Highly efficient mesh-free approach to simulate the non-linear bending analysis of FG porous beams and sandwich beams with FG face sheets", *Thin-Walled Structures*, 185, 110614, 2023.

[11] M. O. Belarbi, M. S. A. Houari, H. Hirane, A. A. Daikh and S. P. A. Bordas, "On the finite element analysis of functionally graded sandwich curved beams via a new refined higher order shear deformation theory", *Composite Structures*, 279, 114715, 2022.

[12] B. Srikarun, W. Songsuwan and N. Wattanasakulpong, "Linear and nonlinear static bending of sandwich beams with functionally graded porous core under different distributed loads", *Composite Structures*, 276, 114538, 2021.

[13] M. Bıˆrsan, T. Sadowski, L. Marsavina, E. Linul and D. Pietras, "Mechanical behavior of sandwich composite beams made of foams and functionally graded materials", *International Journal of Solids and Structures*, 50, 519-530, 2013.

[14] M. Safaei, P. Malekzadeh and M.R. G. Haghighi, "Out-of-plane moving load response and vibrational behavior of sandwich curved beams with GPLRC face sheets and porous core", *Composite Structures*, 327, 117658, 2024.

[15] A. Karamanlı, "Bending behaviour of two directional functionally graded sandwich beams by using a quasi-3d shear deformation theory", *Composite Structures*, 174, 70-86, 2017.