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Solving Problem of Robotic Control of Advanced Missile Via Integral GuptaTransform

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ABSTRACT

A new technique has been brought into play in this article for solving the second-order linear differential equation representing the problem of automatic or robotic control of advanced missiles. The problem of robotic control of the Advanced missile is one of the problems represented by second-order linear differential equations. *T*he Integral Gupta transform (GT) has not at any time been put in an application to solve the problem of robotic control of the advanced missile. In the article, Integral Gupta transform (GT) has been brought into play to solve the problem of robotic control of the advanced missile and reveals that it is a constructive tool for solving such problems. The graphs of the solutions obtained are plotted to indicate the generality and clarity of the proposed method.

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1. INTRODUCTION

The problem of robotic control of the advanced missile is one of the problems represented by a second-order linear differential equation [1]. This paper brings into play the Integral Gupta transform (GT) for solving robotic control of advanced missile problem represented by second order linear differential equation. The mathematical model of problems in Engineering, Physics, Medical, Economics, and Chemistry are constituted by differential equations os various orders [2]. There is a number of techniques for solving these differential equations [3, 6]. Laplace transform scheme has been put in an application extensively for solving problems constituted by differential equations [5]. However, Integral Gupta transform (GT) has not at any time been adequately brought into play in such problems due to its contemporary emergence. The authors Rahul Gupta and Rohit Gupta have proposed the GT in contemporary years to open the door for the affair of solving differential equations [7]. This GT has been already put in an application to solve a horde of initial value problems in intelligence and machination [8, 9].
The GT is outlined as follows:
$\dot{R}\{g(t)\} = G(q) = \frac{1}{q^3}\int_0^\infty e^{-qt}\, g(t)dt,\ t\ \geq 0\,, q_1 \leq q \leq\ q_2$. The labile q is laid out to factor the labile t in the justification of the given function i.e. $g(t)$.

The main motivation for putting in an application the GT for solving *of* problem of robotic control of the advanced missile is that the scheme of solving the governing second-order linear differential equation for such problems is simplified to an algebraic problem. This scheme of recasting the problem of calculus to algebraic problem is specified as operational calculus [10]. The GT scheme has two key benefits over the calculus scheme:

i. A problem ensnaring the differential equation is solved more directly without garnering its general solutions.
ii. A differential equation in non-homogenous form is solved without first solving the related differential equation in homogeneous form.

The GT of particular functions [11] is given as

- $\dot{R}\{t^n\} = \frac{n!}{q^{n+4}}$, $where\ n = 0,1,2,3 \ldots \ldots \ldots$
- $\dot{R}\{sinat\} = \frac{a}{q^3(q^2+a^2)}$, $q > 0$
- $\dot{R}\{cosat\} = \frac{1}{q^2(q^2+a^2)}$, $q > 0$
- $\dot{R}\{e^{at}\} = \frac{1}{q^3(q-a)}$

The GT of particular derivatives [12] is given as

$\dot{R}\{g'(t)\} = qG(q) - \frac{1}{q^3}g(0),$

$\dot{R}\{g''(t)\} = q^2G(q) - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0)$

and so on.

Highlights

- ✓ This article concentrates on the mastery of GT scheme for solving the problem of robotic control of the advanced missile.
- ✓ The differential equation representing the robotic control of the advanced missile problem is solved without first garnering its general solution.
- ✓ Highly refine and logical results are garnered.
- ✓ This article lays out beyond doubt that the GT is a potent mathematical mechanism for solving the problem of robotic control of the advanced missile.

2. RESEARCH METHOD

2.1 Robotic Control of The Ordinary Missile

Assuming that a missile M is searching for a hostile aircraft. If the hostile aircraft turns through some angle $\emptyset(t)$ at time t, then the missile M must also turn through the same angle $\emptyset(t)$, if the missile M has to catch and destroy the hostile aircraft. The second-order linear differential equation representing the **robotic control of the ordinary missile** problem [2-4] is written as

$$I\ddot{\emptyset}(t) + \delta\emptyset(t) = \delta\alpha t$$

Or

$$\ddot{\emptyset}(t) + \frac{\delta}{I}\emptyset(t) = \frac{\delta}{I}\alpha t \quad (1)$$

Here $\emptyset(t)$ is the turning angle of the aircraft at any instant t, $\ddot{\emptyset}(t)$ is the angular acceleration, αt is the assumed desired turning angle of the missile, $\boldsymbol{\alpha}$ is theangular displacemet per unit time of the missile, I is the MOI i.e. moment of inertia and $\delta(> 0)$ is a constant [5, 6].

Further, the initial turning angle is assumed to be $\emptyset(0) = 0$ and initial angular velocity $\dot{\emptyset}(0) = 0$.

The GT of (1) provides

$$q^2\overline{\emptyset}(q) - \frac{1}{q^2}\emptyset(0) - \frac{1}{q^3}\dot{\emptyset}(0) + \frac{\delta}{I}\overline{\emptyset}(q) = \frac{\delta}{I}\alpha\frac{1}{q^5} \quad (2)$$

Here $\overline{\emptyset}(q)$ connotes the GT of $\emptyset(t)$.

Set $\emptyset(0) = \dot{\emptyset}(0) = 0$ and simplifying (2), we get

$$q^2\overline{\emptyset}(q) + \frac{\delta}{I}\overline{\emptyset}(q) = \frac{\delta}{I}\alpha\frac{1}{q^5}$$

$$\Rightarrow \bar{\emptyset}(q) = \frac{\delta\alpha}{I}\{\frac{1}{q^5\left(q^2+\frac{\delta}{I}\right)}\}$$

The above equation can be rewritten as

$$\bar{\emptyset}(q) = \alpha\{\frac{1}{q^5} - \frac{1}{q^3\left(q^2+\frac{\delta}{I}\right)}\}$$

$$\Rightarrow \bar{\emptyset}(q) = \alpha\{\frac{1}{q^5} - \frac{\sqrt{\frac{\delta}{I}}}{\sqrt{\frac{\delta}{I}}\,q^3\left(q^2+\sqrt{\frac{\delta}{I}}^{\,2}\right)}\}$$

Taking inverse GT, we have

$$\emptyset(t) = \alpha\{t - \frac{\sin\sqrt{\frac{\delta}{I}}\,t}{\sqrt{\frac{\delta}{I}}}\}$$

This is the required turn at any instant t.

Considering $\sqrt{\frac{\delta}{I}} = 0.01\ and\ \alpha = 10$, the numerical solution of Equation (1) is shown in figure 1.

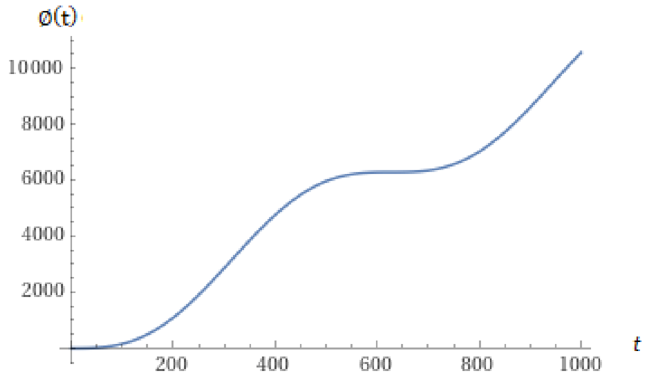

Figure 1: Numerical solution of equation (1)

2.2 Robotic Control of The Advanced Ballistic Missile Problem

The second-order linear differential equation representing the **robotic control of the Advanced Ballistic missile** problem [2-4] is written as

$$t^2 I\ddot{\emptyset}(t) + \delta\emptyset(t) = \delta\alpha t$$

Or

$$t^2\ddot{\emptyset}(t) + \frac{\delta}{I}\emptyset(t) = \frac{\delta}{I}\alpha t \tag{3}$$

The GT of (3) provides

$$q^2\emptyset''(q) + 10q\emptyset'(q) + \left(20+\frac{\delta}{I}\right)\emptyset(q) = \frac{\delta}{I}\alpha\,\frac{1}{q^4}$$

On putting, q = e^x, the above equation is modified to

$$\emptyset''(x) + 9\emptyset'(x) + \left(20+\frac{\delta}{I}\right)\emptyset(x) = \frac{\delta}{I}\alpha e^{-4x} \tag{4}$$

The solution of homogeneous equation: $\emptyset''(x) + 9\emptyset'(x) + \left(20+\frac{\delta}{I}\right)\emptyset(x) = 0$, is given by

$$\emptyset(x) = Ae^{-\frac{1}{2}\left[\sqrt{17-4\frac{\delta}{I}}+1\right]x} + Be^{\frac{1}{2}\left[\sqrt{17-4\frac{\delta}{I}}-1\right]x}$$

$$\emptyset(x) = Ae^{-\,cx} + Be^{fx}$$

$where\ c = \frac{1}{2}\left[\sqrt{17 - 4\frac{\delta}{I}} + 1\right]$ and $f = \frac{1}{2}\left[\sqrt{17 - 4\frac{\delta}{I}} - 1\right]$

The partial integral is given by

$$\text{P.I.}= e^{-\ cx}u + e^{fx}v$$

$$Where\ u = -\int \frac{e^{fx}\frac{\delta}{I}\alpha e^{-4x}}{[e^{-\ cx}\frac{d}{dx}e^{fx} - e^{fx}\frac{d}{dx}e^{-cx}]}dx = -\frac{\delta}{I}\frac{\alpha e^{(-4+c)x}}{(c+f)(-4+c)}$$

$$And\ v = \int \frac{e^{-cx}\frac{\delta}{I}\alpha e^{-4x}}{[e^{-\ cx}\frac{d}{dx}e^{fx} - e^{fx}\frac{d}{dx}e^{-cx}]}dx = \frac{\delta}{I}\frac{\alpha e^{(-4-f)x}}{(c+f)(4+f)}$$

$$\text{P.I.}= e^{-\ cx}u + e^{fx}v = -\frac{\delta}{I}\frac{\alpha e^{-4x}}{(c+f)(-4+c)} + \frac{\delta}{I}\frac{\alpha e^{-4x}}{(c+f)(4+f)} = -\frac{\delta\alpha}{I(c+f)}\left[\frac{1}{(-4+c)} - \frac{1}{(4+f)}\right]e^{-4x} = \frac{7\alpha}{(8+\frac{\delta}{I})\sqrt{17-4\frac{\delta}{I}}}e^{-4x}$$

Thus, the solution of equation (4) is given by

$$\emptyset(\text{x}) = Ae^{-\ cx} + Be^{fx} + \frac{7\alpha}{(8+\frac{\delta}{I})\sqrt{17-4\frac{\delta}{I}}}e^{-4x}$$

Replacing x by logq, we have

$$\emptyset(\text{q}) = Ae^{-\ c\log q} + Be^{f\log q} + \frac{7\alpha}{(8+\frac{\delta}{I})\sqrt{17-4\frac{\delta}{I}}}e^{-4\log q}$$

$$\emptyset(\text{q}) = Aq^{-\ c} + Bq^{f} + \frac{7\alpha}{(8+\frac{\delta}{I})\sqrt{17-4\frac{\delta}{I}}}\frac{1}{q^4} \quad (5)$$

Taking inverse GT, we have

$$\emptyset(\text{t}) = \frac{At^{-c-4}}{\Gamma(-c-3)} + \frac{Bt^{f-4}}{\Gamma(f-3)} + \frac{7\alpha}{(8+\frac{\delta}{I})\sqrt{17-4\frac{\delta}{I}}}t \quad (6)$$

To find the constants A and B, we will apply conditions: $\emptyset(1) = 0\ and\ \emptyset'(1) = 0.$
On applying these conditions to equaton (6) and solving, we obtain:

$$A = \frac{7\alpha(5-f)\Gamma(-c-3)}{(8+\frac{\delta}{I})(17-4\frac{\delta}{I})}\ and\ B = \frac{-7\alpha(5+c)\Gamma(f-3)}{(8+\frac{\delta}{I})(17-4\frac{\delta}{I})}$$

Hence equation (6) becomes

$$\emptyset(\text{t}) = \frac{7\alpha(5-f)}{(8+\frac{\delta}{I})(17-4\frac{\delta}{I})}t^{-c-4} - \frac{7\alpha(5+c)}{(8+\frac{\delta}{I})(17-4\frac{\delta}{I})}t^{f-4} + \frac{7\alpha}{(8+\frac{\delta}{I})\sqrt{17-4\frac{\delta}{I}}}t$$

$$\emptyset(\text{t}) = \frac{7\alpha}{(8+\frac{\delta}{I})\sqrt{17-4\frac{\delta}{I}}}t + \frac{7\alpha}{2}\frac{\left(7-\sqrt{17-4\frac{\delta}{I}}\right)}{(8+\frac{\delta}{I})(17-4\frac{\delta}{I})}t^{-\frac{1}{2}\left[\sqrt{17-4\frac{\delta}{I}}+9\right]} - \frac{7\alpha}{2}\frac{\left(7+\sqrt{17-4\frac{\delta}{I}}\right)}{(8+\frac{\delta}{I})(17-4\frac{\delta}{I})}t^{\frac{1}{2}\left[\sqrt{17-4\frac{\delta}{I}}-9\right]} \quad (7)$$

This is the required turn at any instant t.
Considering $\frac{\delta}{I} = 0.01\ and\ \alpha = 10$, the numerical solution of Equation (3) is shown in figure 2.

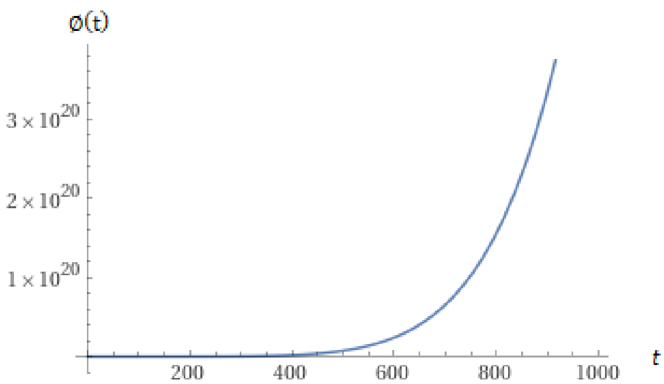

Figure 2: Numerical solution of equation (3)

3. RESULTS

The problem of robotic control of the advanced missile represented by the second-order linear differential equation has been solved successfully by laying out the Integral Gupta transform (GT). It lays out that it is doable to put in an application the GT as a solving technique for solving the problem of robotic control of the advanced missile.

4. CONCLUSION

A novel method, that is, integral GT has been exploited successfully for solving the problem of robotic control of the advanced missile represented by the second-order linear differential equation. This paper tenders an alternate style for solving the problem of robotic control of the advanced missile.

FUTURE SCOPE

In near future, GT is expected to be applied to fractional differential equations and in cryptography.

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