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Approximated transverse deflection of sandwich beam with 2D-FG and ceramic face sheets and 1D-FG core

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ABSTRACT

The approximated numerical deflection of a sandwich beam with two directional functionally graded (2D-FG) and ceramic face sheets and one directional functionally graded (1D-FG) core, namely SW2D1DC, is presented under uniform load and various boundary conditions. The finite element code written in Matlab is applied in this article to investigate the influences of material properties on transverse deflections. The results of this article are given and compared with other results in the references to verify the feasibility of the application. This study also provides some more information about the characteristics of SW2D1DC beams.

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1. INTRODUCTION

Nowadays, functionally graded material has become one of the smart materials, and it is widely used in many different fields. For example, lots of production related to the defense industry, like nuclear tanks, spacecraft, etc. are produced based on the above material. Due to the high applicability of functionally graded material, many studies related to various theories have been given to comment the mechanical behavior of functionally graded structures such as [1-3]. However, porosity of the material can occur during the manufacturing process [4, 5]. So, a study related to this issue must be considered as soon as possible to have a good knowledge of the porosity effect on mechanical behavior of structures. There are three types of structure, like beam, plate and shell, but researchers are usually interested in beam structures because of their wide applications. Using a simple Timoshenko beam model [6-8] helps us to reduce the computational cost with the resulting error within the allowable range. On the other hand, sandwich beams made of functionally graded porous materials should be investigated as much as possible to help the designer have the right knowledge about the mechanical properties. The few published papers on bending behavior of sandwich beams can be listed here [9-15]. In the investigation of [9], linear and nonlinear bending analyses of sandwich beams with functionally graded cores were determined under different types of distributed loads. These sandwich beams were composed of two isotropic faces and a porous core with different gradients of internal pores. The governing formulation used to describe the beam's linear and nonlinear behavior was constructed from Reddy's third-order shear deformation theory and nonlinear strain–displacement relations of von Kármán. The Gram-Schmidt orthogonalization procedure was adopted to generate numerically stable functions for the displacement field to solve the beam problems with various boundary conditions. Then, the

Ritz method was utilized to find out linear and nonlinear bending results in conjunction with the iterative technique. The paper [10] was devoted to a sandwich beam with an asymmetric structure under a uniformly distributed load. The first end of the beam was simply supported, while the second end was clamped. The individual nonlinear deformation function of a plane beam cross-section – the shear effect, with consideration of the classical shear stress formula, was analytically developed. Displacements, strains and stresses, taking into account the individual nonlinear deformation function for successive layers of this beam were elaborated in detail. The system of two differential equations of equilibrium of this beam, based on the principle of stationary potential energy, was obtained. This system of equations was precisely solved analytically, according to the theory of differential equations. The authors in [11] investigated sandwich composite beams using a direct approach which models slender bodies as deformable curves endowed with a certain microstructure. They derived general formulas for the effective stiffness coefficients of composite elastic beams made of several non-homogeneous materials. A special attention was given to sandwich beams with foam core, which were made of functionally graded or piecewise homogeneous materials. In the case of small deformations, the theoretical predictions were compared with experimental measurements for the three-point bending of sandwich beams, showing very good agreement. For functionally graded sandwich columns, they obtained the analytical solutions of bending, torsion and extension problems and compared them with numerical results computed by the finite element method. The main objective of the work in [12] was to propose a coupling of the radial point interpolation method with variable shape parameter to high order continuation method for analyzing the geometrically non-linear behavior of porous functionally graded isotropic and sandwich Timoshenko beams by taking into account the non-linear shear deformation. The functionally graded and functionally graded sandwich beams effective material properties were assumed to vary continuously in the thickness direction according to a power-law index. The porosity was modeled as the stiffness reduction criteria and included in the mixture rule. The strong form of the non-linear equations governing porous functionally graded and functionally graded sandwich beams beams were obtained using the principle of virtual displacements in the framework of first-order shear deformation theory. The high order continuation method was used to compute the solution of the non-linear problem by a transformation to recursive succession of linear problems using Taylor series expansion. The continuation technique was used to obtain the complete solution path in a step by step manner. As a first endeavor, the out-of-plane free vibrational behaviors and moving load responses of sandwich curved beams with graphene platelets reinforced composite face sheets and porous core were carried out in [13]. The governing equations were derived based on the first-order shear deformation theory using Hamilton's principle, which were discretized by employing the differential quadrature method and Newmark's method in the spatial and time domains, respectively. To simulate the moving load using the differential quadrature method, the Heaviside function approach was utilized. The effective elastic properties of face sheets were estimated using Halpin-Tsai micromechanical model. The approach was validated by presenting convergence studies and accuracy verification followed by parametric studies. The numerical results indicated that by adding little amount of graphene platelets into the face sheets and core layer, the fundamental natural frequency and the displacement amplitudes under moving load significantly increased and decreased, respectively, regardless of the beam boundary conditions. In the paper [14], a novel refined shear deformation beam theory was proposed and applied, for the first time, to investigate the bending behavior of functionally graded sandwich curved beam. The theory was exploited to satisfy parabolic variation of shear stress distribution along the thickness direction thereby obviating the use of any shear correction factors. The material properties of functionally graded sandwich beam changed continuously from one surface to another according to a power-law function. Three common configurations of FG beams were considered for the study, namely: single layer functionally graded beam; sandwich beam with functionally graded face sheets and homogeneous core and sandwich beams with homogeneous face sheets and functionally graded core. The governing equations derived herein were solved by employing the finite element method using a two-noded beam element. The paper [15] presented the static behaviour of two-directional functionally graded sandwich beams subjected to various sets of boundary conditions by using a quasi-3D shear deformation theory and the Symmetric Smoothed Particle Hydrodynamics method. The Symmetric Smoothed Particle Hydrodynamics code, which was developed based on the present formulation of the functionally graded sandwich beam, was validated by solving a simply supported conventional functionally graded beam problem. The response and energy absorption of novel sandwich beams with combined re-entrant double-arrow auxetic honeycomb cores subjected to three-point bending were studied experimentally and numerically by [16]. Two typical sandwich beams loaded at different loading positions were considered. Quasi-static three-point bending experiments were conducted to obtain the failure modes and force–displacement curves. The reliable numerical simulation models were further established based on experimental validations. Although the auxetic structure exhibits excellent mechanical properties, the low stiffness, and strength still limitted its development. Considering the high stiffness and strength of the sandwich structure, a novel sandwich beam with an enhanced auxetic core was proposed in paper [17]. The accuracy of the finite element modeling was first verified by quasi-static bending experiments. Then, based on this simulation, the deformation modes and energy absorption were explored. The results showed that local indentation and global bending deformation coexisted under the mid-span loading, and the core can absorb the most energy. A typical sandwich composite beam included a soft core covered by two stiff face layers was introduced in [18]. For such beam structures, large stiffness difference between two adjacent layers could result in shear deformations and zigzag displacement phenomenon. In the [18], a novel higher-order refined zigzag theory was presented for solving the static bending problems of a sandwich composite beam with a soft core. This higher-order refined zigzag theory was derived based on the refined zigzag theory by adding the higher-order zigzag terms. The authors in [19] had recently developed a porous hydroxyapatite / poly(lactide-co-ε-caprolactone) composite beam as a new scaffold material prospectively used in the field of bone tissue engineering. In this article, a novel sandwich beam structure was introduced to improve the bending mechanical properties of the single hydroxyapatite / poly(lactide-co-ε-caprolactone) beam. The middle hydroxyapatite / poly(lactide-co-ε-caprolactone) layer with the two-phase porous structure was sandwiched between the top and bottom spongy poly(lactide-co-ε-caprolactone) layer. It was revealed from the results of three-point bending tests that the bending mechanical properties of the composite sandwich beam were significantly higher than others. In the work of [20], bending and free vibration analyses of functionally graded carbon nanotube-reinforced sandwich beams were carried out using finite element-based higher-order zigzag theory. Face sheets were assumed to be made up of functionally graded carbon nanotube-reinforced composite, and the core was assumed to be made up of balsa wood. The computational model incorporated transverse shear stress and transverse normal stress continuity condition at interfaces. Zero transverse shear stress condition at the bottom and top surfaces of the beam was also satisfied. The principle of minimum potential energy was employed for carrying out bending analysis, respectively. In the study of [21], sandwich beams with hierarchical honeycomb cores were proposed. Corresponding mechanical responses subjected to three-point bending load were explored and compared with that of the sandwich equipped with conventional honeycomb core by means of numerical simulation. Theoretical solutions of equivalent bending stiffness coincided well with numerical results. Metallic sandwich structures with pyramidal cellular cores had proven to be highly effective in the industry of advanced lightweight materials thanks to their high strength-to-weight ratios. The paper [22] presented the investigation of the bending performance of a sandwich structure based on a novel pyramidal cellular core. The effective bending and shear stiffness were investigated using numerical and experimental approaches. Furthermore, the finite element model created for the four-point bending scenario was in good agreement with the experimental one and could be subsequently used for further product development. The study [23] presented a new method for forming the graded corrugated truss core sandwich structures based on an auto-cutting and mould-press process. The bending stiffness, strength and failure mechanism were investigated. Analytical models were presented to estimate the performance and failure mode of the sandwich beams. In order to demonstrate sensitivity of geometric parameters on the bending behavior of the truss core sandwich beams, a uniform and two kinds of graded corrugated truss core sandwich panels were fabricated and tested to probe different failure modes. The bending–torsion coupling for a sandwich beam was analysed from the static and vibrational point of view by [24]. Off-axis orientations in the skin and transverse graded core were proposed to perform the coupling. Solutions were first compared numerically from a static point of view using a complete beam theory. The natural frequencies and mode shapes were then studied experimentally and numerically. Applications of sandwich structures, comprising alumina face sheets and aluminum foam core, depended critically on their mechanical performance. Four point bend tests were performed on sandwich beams with varying geometries to identify competing failure modes, such as core indentation, face sheet cracking and core shear as in [25]. Analytical formulae for the identified failure modes were obtained. A failure mode map was constructed based on the analytical calculations in the non-dimensional parameters of beam geometry for a given face sheet to core strength ratio, etc. Returning to this article, the numerical results obtained are completely based on the simple beam model, so the algorithm construction process will be light and does not require much computational cost. The small error when compared with the results of other documents is a clear demonstration of the feasibility of this way.

This article is constructed in four sections. Section 1 gives the introduction about sandwich beams as above. Section 2 presents the formulation for sandwich beam with 2D-FG and ceramic face sheets and 1D-FG core and then Section 3 shows some numerical results. Finally, a few comments are also given in Section 4 respectively.

2. FORMULATION

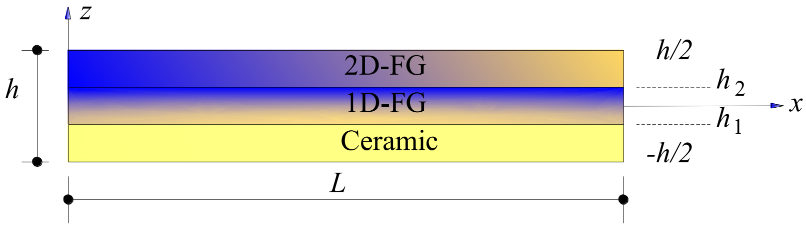

Figure 1. Sandwich beam with 2D-FG and ceramic face sheets and 1D-FG core

$$\begin{cases} M(z) = M_c, \quad -h/2 \le z \le h_1 \\ M(z) = M_m + (M_c - M_m)\left(\dfrac{z - h_2}{h_1 - h_2}\right)^{n_z}, \quad h_1 < z < h_2 \\ M(z) = M_m + (M_c - M_m)\left(\dfrac{z - h_2}{h/2 - h_2}\right)^{n_z}\left(1 - \dfrac{x}{2L}\right)^{n_x}, \quad h_2 \le z \le h/2 \end{cases} \tag{1}$$

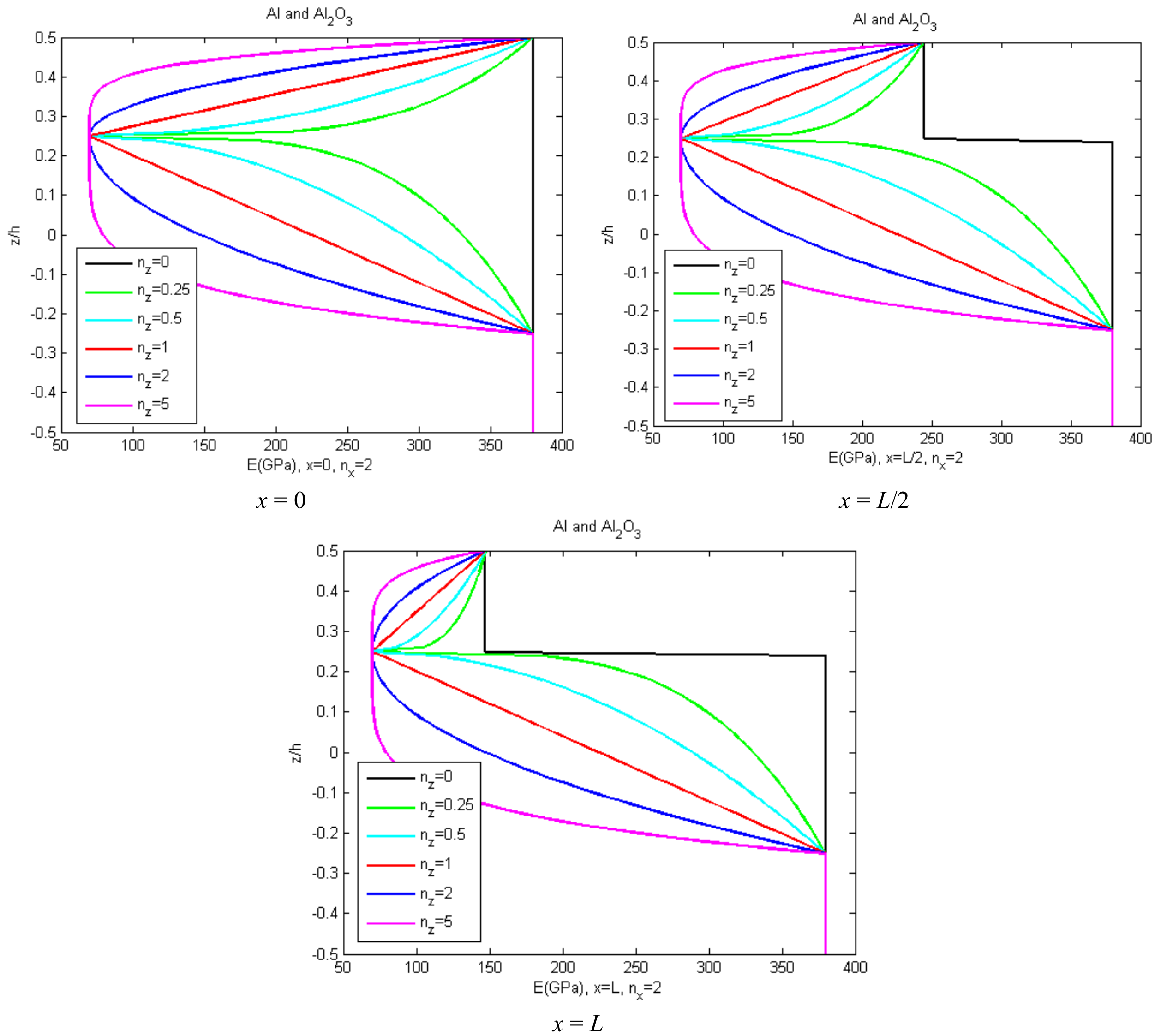

Figure 2. The modification of E with $n_x = 2$ and ratio of the thicknesses [*1/2/1*]

A SW2D1DC beam of length L and thickness h is studied. This beam consists of one homogenous ceramic face sheet, two directional functionally graded face sheet and one directional functionally graded core as shown in Figure 1. The values of material properties $M(z)$ are formulated in Equation (1). Besides, a group of three numbers like "z_1 / z_2 / z_3" is used to denote the ratio of the thicknesses of the bottom-core-top layers. It means the thickness of the bottom layer is $h.z_1 / (z_1 + z_2 + z_3)$, that of the core layer is $h.z_2 / (z_1 + z_2 + z_3)$ and that of the top layer is $h.z_3 / (z_1 + z_2 + z_3)$. The effective Young's modulus is demonstrated in Figures 2 with beam contains Al and Al_2O_3 materials. Based on finite element procedure, the stiffness matrix of an element of beam can be written

$$\mathbf{K}_e = \frac{E_e I_e}{L_e^3\left(1+\Phi\right)}\begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & \left(4+\Phi\right)L_e^2 & -6L_e & \left(2-\Phi\right)L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & \left(2-\Phi\right)L_e^2 & -6L_e & \left(4+\Phi\right)L_e^2 \end{bmatrix}, \quad \Phi = \frac{12E_e I_e}{G_e k A_e L_e^2}, \quad k = 5/6 \tag{2}$$

The element equation is given

$$\mathbf{K}_e \times \begin{bmatrix} w_i & \varphi_i & w_j & \varphi_j \end{bmatrix}^T = \begin{bmatrix} f_i & m_i & f_j & m_j \end{bmatrix}^T \tag{3}$$

After assembly, the deflections can be obtained by solving the following equation

$$\mathbf{Kd} = \mathbf{F} \tag{4}$$

By using two letters 'C' and 'S' refer to the clamped and simply supported condition, two kinds of boundary conditions can be taken as (SS) and (CC) for SW2D1DC beams in this article.

3. NUMERICAL RESULTS

Firstly, the verification of the proposed model is presented for (SS/CC) SW2D1DC beam with $L/h = 5$, $n_x = 0$ and $n_z = 0$ under a uniform load q. The material properties of the beam are provided in Table 1.

Table 1. The material properties

Properties	Metal: Al	Ceramic: Al_2O_3
E (GPa)	70	380
ν	0,3	0,3

The maximum deflection is normalized by $\bar{w} = 100E_m h^3 w(L/2)/q/L^4$. The values of dimensionless deflections are given in Table 2 and compared with other results from another beam theory in [15]. It can be seen that the results obtained from this article are approximate with other results related to analytical method (Vo et al.) or symmetric smoothed particle hydrodynamics method (Karamanli).

Table 2. The comparison of the dimensionless deflections of (SS) SW2D1DC beams

BCs	Method		
	Vo et al.	Karamanli	Present
SS	3.1397	3.1425	3.1060
CC	0.8327	0.8421	0.8488

The next problem will be developed by investigating the variation of the maximum deflection when changing the parameters n_x, n_z, L/h ratio and the ratio of the thicknesses for (SS) SW2D1DC beams. The obtained numerical results are plotted in Figure 3. It is easy to see that increasing the value of n_x or n_z leads to an increase in the deflection value of the beam in all cases.

Finally, by changing different boundary condition from (SS) to (CC), numerical results are obtained quite quickly as illustrated in Figure 4, respectively.

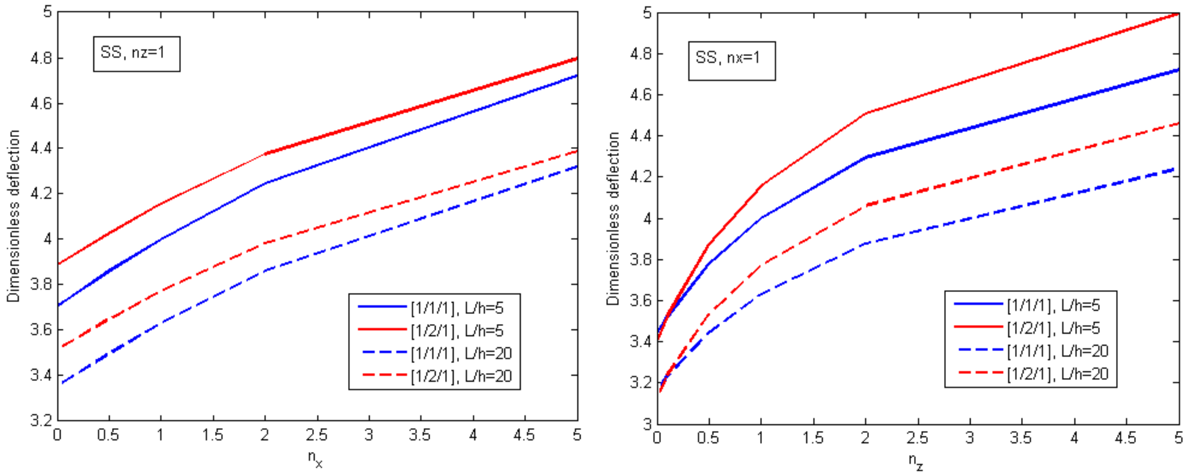

Figure 3. The approximated dimensionless deflections at $x = L/2$ for (*SS*) SW2D1DC beams under changing n_x or n_z

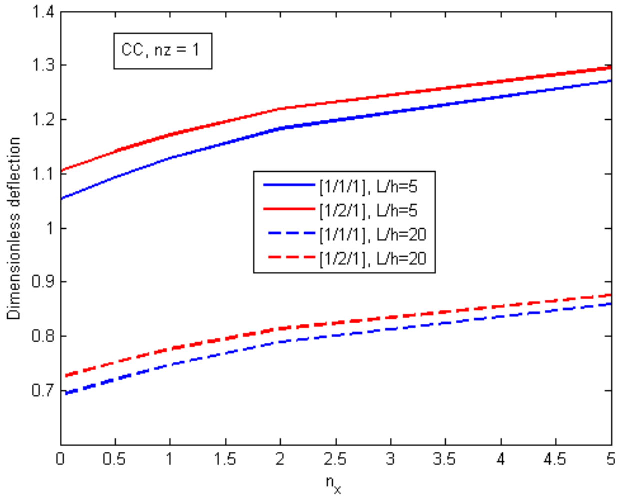

Figure 4. The approximated dimensionless deflections at $x = L/2$ for (*CC*) SW2D1DC beams

4. CONCLUSION

In this article, the transverse deflections of SW2D1DC beams under two different types of boundary condition: *SS & CC* are given. The proposed results are approximate with others in references. The main aim is to affirm the applicability of a simple beam model to analyze the SW2D1DC beams with acceptable results.

REFERENCES

[1] D. D. Singh, *et al.*, "Functionally graded materials manufactured by direct energy deposition: A review", *Materials Today: Proceedings*, vol. 47, pp. 2450-2456, 2021.
[2] A. Pasha and B. M. Rajaprakash, "Fabrication and mechanical properties of functionally graded materials: A review", *Materials Today: Proceedings*, vol. 52, pp. 379-387, 2022.
[3] E. J. Kim, *et al.*, "Study on the characteristics of functionally graded materials from Ni-20cr to Ti-6Al-4V via directed energy deposition", *Journal of Alloys and Compounds*, vol. 955, p. 170263, 2023.
[4] D. Khrapov, *et al.*, "Geometrical features and mechanical properties of the sheet-based gyroid scaffolds with functionally graded porosity manufactured by electron beam melting", *Materials Today Communications*, vol. 35, p. 106410, 2023.
[5] Y. Xiong, *et al.*, "Effects of porosity gradient pattern on mechanical performance of additive manufactured Ti-6Al-4V functionally graded porous structure", *Materials & Design*, vol. 208, p. 109911, 2021.
[6] E. Ruocco and J.N. Reddy, "Analytical solutions of Reddy, Timoshenko and Bernoulli beam models: A comparative analysis", *European Journal of Mechanics - A/Solids*, vol. 99, p. 104953, 2023.
[7] H. Qin, *et al.*, "Modified Timoshenko beam model for bending behaviors of layered materials and structures", *Extreme Mechanics Letters*, vol. 39, p. 100799, 2020.

[8] Y. Kaya and J. Dowling, "Application of Timoshenko beam theory to the estimation of structural response", *Engineering Structures*, vol. 123, pp. 71-76, 2016.
[9] B. Srikarun, *et al.*, "Linear and nonlinear static bending of sandwich beams with functionally graded porous core under different distributed loads", *Composite Structures*, vol. 276, p. 114538, 2021.
[10] K. Magnucki and E. M. Blandzi, "A refined shear deformation theory of an asymmetric sandwich beam with porous core: Linear bending problem", *Applied Mathematical Modelling*, vol. 124, pp. 624-638, 2023.
[11] M. Bıˆrsan, *et al.*, "Mechanical behavior of sandwich composite beams made of foams and functionally graded materials", *International Journal of Solids and Structures*, vol. 50, pp. 519-530, 2013.
[12] S. Mesmoudi, *et al.*, "Highly efficient mesh-free approach to simulate the non-linear bending analysis of FG porous beams and sandwich beams with FG face sheets", *Thin-Walled Structures*, vol. 185, p. 110614, 2023.
[13] M. Safaei, *et al.*, "Out-of-plane moving load response and vibrational behavior of sandwich curved beams with GPLRC face sheets and porous core", *Composite Structures*, vol. 327, p. 117658, 2024.
[14] M. O. Belarbi, *et al.*, "On the finite element analysis of functionally graded sandwich curved beams via a new refined higher order shear deformation theory", *Composite Structures*, vol. 279, p. 114715, 2022.
[15] A. Karamanlı, "Bending behaviour of two directional functionally graded sandwich beams by using a quasi-3d shear deformation theory", *Composite Structures*, vol. 174, pp. 70-86, 2017.
[16] H. Wang, *et al.*, "Three-point bending response and energy absorption of novel sandwich beams with combined re-entrant double-arrow auxetic honeycomb cores", *Composite Structures*, vol. 326, p. 117606, 2023.
[17] H. Lu, *et al.*, "Quasi-static bending response and energy absorption of a novel sandwich beam with a reinforced auxetic core under the fixed boundary at both ends", *Thin-Walled Structures*, vol. 191, p. 111011, 2023.
[18] C. D. Chen and B. F. Huang, "A novel higher-order refined zigzag theory for static bending analysis in sandwich composite beam", *Applied Mathematical Modelling*, vol. 119, pp. 586-604, 2023.
[19] F. Hazwani and M. Todo, "Deformation mechanism of porous composite sandwich beam for orthopaedical application under three-point bending", *Composite Structures*, vol. 281, p. 114983, 2022.
[20] A. Garg, *et al.*, "Bending and free vibration analysis of symmetric and unsymmetric functionally graded CNT reinforced sandwich beams containing softcore", *Thin-Walled Structures*, vol. 170, p. 108626, 2022.
[21] Z. Li, *et al.*, "Bending behavior of sandwich beam with tailored hierarchical honeycomb cores", *Thin-Walled Structures*, vol. 157, p. 107001, 2020.
[22] M. Iftimiciuc, *et al.*, "Bending performance of a sandwich beam with sheet metal pyramidal core", *Materials Today Communications*, vol. 31, p. 103490, 2022.
[23] G. D. Xu, *et al.*, "Bending behavior of graded corrugated truss core composite sandwich beams", *Composite Structures*, vol. 138, pp. 342-351, 2016.
[24] L. Bernard, *et al.*, "Static and dynamic analysis of bending–torsion coupling of a CFRP sandwich beam", *Composite Structures*, vol. 145, pp. 26-36, 2016.
[25] K. Mohan, *et al.*, "Failure of sandwich beams consisting of alumina face sheet and aluminum foam core in bending", *Materials Science and Engineering: A*, vol. 409, pp. 292-301, 2005.