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Response of Non-Damped Oscillators Subjected To Rectangular Pulse

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ABSTRACT

In this manuscript, a new integral transform, that is, Rohit transform (RT) is put to use for finding the response of non-damped oscillators (i.e. electrical and mechanical oscillators) subjected to a rectangular pulse. Generally, this problem has been treated by methods like calculus or Laplace transforms. Also, some operational properties of the integral RT are discussed. This manuscript put forward a novel technique, that is, RT for finding the response of non-damped oscillators subjected to a rectangular pulse. It is found that before the removal of rectangular pulse, the nature of response of non-damped mechanical oscillators subjected to a rectangular pulse increases and decreases periodically with constant amplitude but when the rectangular pulse is removed, the nature of response becomes oscillatory with constant amplitude. The results fixed are the same as fixed with other methods. This paper tenders an alternate style for finding the response of non-damped oscillators (i.e. electrical and mechanical oscillators) subjected to a rectangular pulse.

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1. INTRODUCTION

The rectangular pulse force (shown in the figure) is written as [1], [2], [3]:

$$F(t) = F_o \; for \; t < t_1$$
$$= 0 \; for \; t \geq t_1.$$

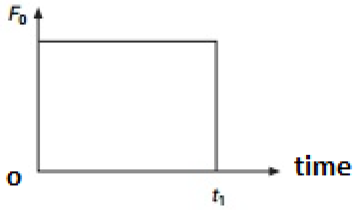

Figure: Rectangular pulse function

The Rohit transform of a function $g(t)$ [4], [5] is defined by the integral equations as:

$$R\{g(t)\} = q^3 \int_0^\infty e^{-qt}\, g(t)dt,\ t \geq 0\ , q_1 \leq q \leq q_2.$$

The variable q in this transform is used to factor the variable t in the argument of the function g. The integral RT has been submitted by Rohit Gupta in contemporary years and put in to unfold the problems in engineering, science and technology. The key motivation for applying RT for finding the response of non-damped oscillators (i.e. electrical and mechanical oscillators) subjected to a rectangular pulse is that the process of solving a governing ordinary differential equation for such problem is simplified to an algebraic problem. This method of converting the problems of calculus to algebraic problems is known as operational calculus. The RT, when applied to a function, changes that function into a new function by using a process that involves integration. The RT method has two main advantages over the calculus method:

- **i.** Problems involving differential equations are worked out more directly i.e. initial (boundary) value problems are worked out without first ascertaining a general solution.
- **ii.** A Non-homogenous differential equation is worked out without first working out the corresponding homogeneous differential equation.

The RT of a few elementary functions [6] is given by

- ❖ $R\,\{t^n\} = \frac{n!}{q^{n-2}}, where\ n\ is\ 0, 1, 2\ ....$
- ❖ $R\,\{sinbt\} = \frac{b\,q^3}{q^2+b^2},$
- ❖ $R\,\{cosbt\} = \frac{q^4}{q^2+b^2},$

A unit step function is written as $U(t-a) = 0\ for\ t < a\ and\ 1\ for\ t \geq$ a.
The RT of a unit step function is given by

$$R\{U(t-a)\} = q^3 \int_0^\infty e^{-qt}\, U(t-a)dt$$

$$R\{U(t-a)\} = q^3 \int_a^\infty e^{-qt}\, dt$$

$$R\{U(t-a)\} = q^2 e^{-qa}$$

Shifting property:
If $R\{g(t)\} = G(q),\ then\ R[g(t-a)U(t-a) = e^{-qa}G(q).$
Proof:

$$R[g(t-a)U(t-a) = q^3 \int_0^\infty e^{-qt}\, g(t-a)U(t-a)dt$$

$$R[g(t-a)U(t-a) = q^3 \int_a^\infty e^{-qt}\, g(t-a)dt$$

$$R[g(t-a)U(t-a) = q^3 \int_0^\infty e^{-q(v+a)}\, g(v)dv, \quad where\ v = t-a$$

$$R[g(t-a)U(t-a) = e^{-q(a)}\, q^3 \int_0^\infty e^{-q(v)}\, g(v)dv$$

$$R[g(t-a)U(t-a) = e^{-q(a)}\, q^3 \int_0^\infty e^{-q(t)}\, g(t)dt$$

$$R[g(t-a)U(t-a) = e^{-q(a)}G(q)$$

The RT of a few derivatives [6] is given by
$R\,\{g'(t)\} = qR\{g(t)\} - q^3 g(0),$
$R\{g''(t)\} = q^2 R\{g(t)\} - q^4 g(0) - q^3 g'(0),$ and so on.

Highlights

- ✓ Focusing on any research effort concerning literature review is the very paramount task because it builds up ideas that can evolve quickly.This paper focuses on the administration of RT an upto the minute integral transform technique.
- ✓ An RT technique is proposed for handing out the response of non-damped oscillators subjected to a rectangular pulse.

The differential equation representing the non-damped oscillators subjected to a rectangular pulse is worked out more directly without first ascertaining a general solution.

- ✓ A non-homogenous differential equation representing the non-damped oscillators subjected to a rectangular pulse is worked out without first working out the corresponding homogeneous differential equation.
- ✓ Graphs of responses with respect to time are plotted.
- ✓ Highly precise and accurate results are obtained.
- ✓ This paper shows beyond doubt that the integral transform RT is a potent mathematical tool for analyzing the non-damped oscillators subjected to a rectangular pulse.

2. RESEARCH METHOD

The article is outlined as: First, a brief inception of the RT is laid out. Second, the enactment of the RT to non-damped oscillators (i.e. electrical and mechanical oscillators) subjected to a rectangular pulse is explained. Finally, the argumentation and the deduction are furnished.

2.1 Non-damped Mechanical Oscillator subjected to a rectangular pulse force

The response of a non-damped mechanical oscillator [7], [8] subjected to a rectangular pulse force is obtained from the following equation

$$m\ddot{y}(t) + ky(t) = F(t)$$

Or

$$\ddot{y}(t) + \omega_0{}^2 y(t) = \frac{F(t)}{m} \quad \ldots. (1)$$

$where\ \omega_0 = \sqrt{\frac{k}{m}}$, F(t) is a rectangular pulse force. Also, [9] y (0) = 0 & $\dot{y}(0) = 0$.

The RT of (1) provides

$$q^2\bar{y}(q) - q^4 y(0) - q^3\dot{y}(0) + \omega_0{}^2\bar{y}(q) = \frac{1}{m}q^3 \int_0^\infty e^{-qt}\, F(t)\, dt$$

$$\Rightarrow q^2\bar{y}(q) - q^4 y(0) - q^3\dot{y}(0) + \omega_0{}^2\bar{y}(q) = \frac{F_o}{m}q^3 \int_0^{t_1} e^{-qt}\ dt + \frac{1}{m}\ q^3 \int_{t_1}^\infty e^{-qt}\,(0) dt$$

Here $\bar{y}(q)$ is the RT of y(t).

Put $y(0) = 0$ and $\dot{y}(0) = 0$, we get

$$q^2\bar{y}(q) + \omega_0{}^2\bar{y}(q) = \frac{F_o}{m}q^3 \int_0^{t_1} e^{-qt}\ dt$$

$$\Rightarrow q^2\bar{y}(q) + \omega_0{}^2\bar{y}(q) = -q^2\ \frac{F_o}{m}[e^{-qt_1} - 1]$$

$$\Rightarrow q^2\bar{y}(q) + \omega_0{}^2\bar{y}(q) = \frac{F_o}{m}\{q^2 - q^2 e^{-qt_1}\}$$

$$\Rightarrow \bar{y}(q) = \frac{F_o}{m}\{\frac{q^2}{(q^2 + \omega_0{}^2)} - \frac{q^2}{(q^2 + \omega_0{}^2)}e^{-qt_1}\}$$

$$\Rightarrow \bar{y}(q) = \frac{F_o}{m}\{\frac{q^4}{q^2(q^2 + \omega_0{}^2)} - \frac{q^4}{q^2(q^2 + \omega_0{}^2)}e^{-qt_1}\}$$

$$\Rightarrow \bar{y}(q) = \frac{F_o}{m}\{\frac{q^2}{(\omega_0{}^2)} - \frac{q^4}{(\omega_0{}^2)(q^2 + \omega_0{}^2)} - \frac{q^2}{(\omega_0{}^2)}e^{-qt_1} + \frac{q^4}{(\omega_0{}^2)(q^2 + \omega_0{}^2)}e^{-qt_1}\}$$

Taking inverse RT, we have

$$y(t) = \frac{F_o}{m}\{\frac{1}{(\omega_0{}^2)} - \frac{\cos\omega_0 t}{(\omega_0{}^2)} - \frac{1}{(\omega_0{}^2)}U(t - t_1) + \frac{\cos\omega_0(t - t_1)}{(\omega_0{}^2)}U(t - t_1)\}$$

$$\Rightarrow y(t) = \frac{F_o}{m(\omega_0{}^2)}\ \{1 - \cos\omega_0 t - U(t - t_1) + \cos\omega_0 t\, U(t - t_1)\}$$

$$\Rightarrow y(t) = \frac{F_o}{k}\ \{1 - \cos\omega_0 t - U(t - t_1) + \cos\omega_0 t\, U(t - t_1)\} .. (3)$$

For t < t_1,

$y(\mathrm{t}) = \frac{F_o}{k}\{ 1 - \cos \omega_0 t\} \ldots \ldots (4a)$

Taking F_o = 100N, k = 1000N/m and ω_0=314rad/s, the graphs of (4a) is shown in the figure 1.

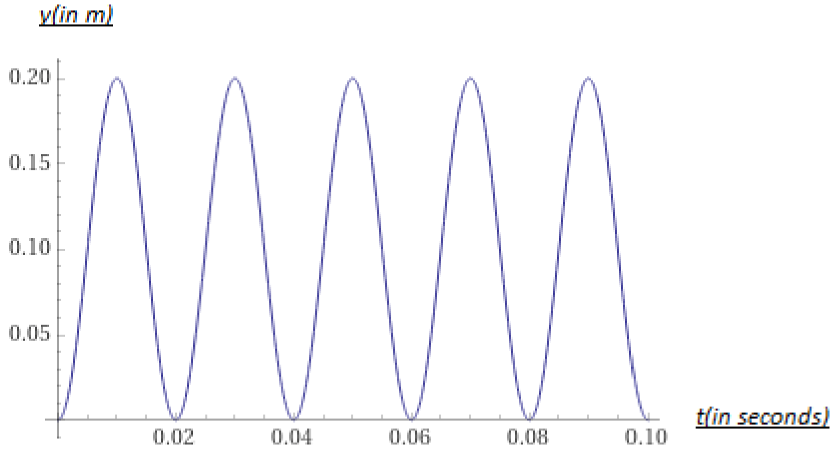

Figure 1: Respone of Non-damped Mechanical Oscillator before removal of rectangular pulse force

For t > t_1,

$y(\mathrm{t}) = \frac{F_o}{k}\{ -\cos \omega_0 t + \cos \omega_0 (t - t_1)\} \ldots \ldots$ (4b)

Taking F_o = 100N, k = 1000N/m, t_1 = 0.1s and ω_0=314rad/s, the graphs of (4b) is shown in the figure 2.

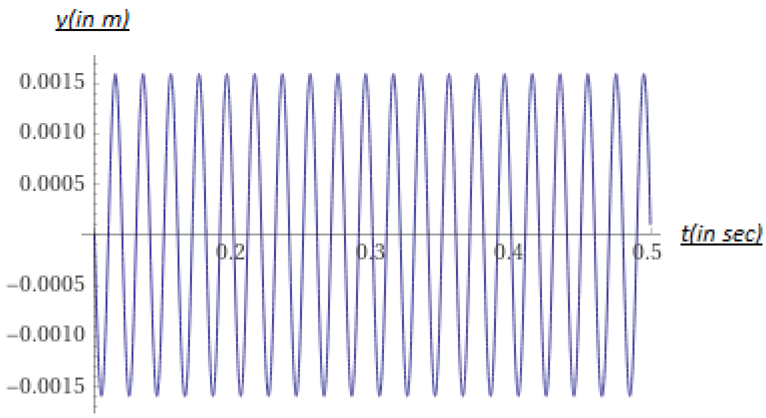

Figure 2: Respone of Non-damped Mechanical Oscillator after removal of rectangular pulse force

2.1 Non-damped Electrical Oscillator subjected to a rectangular pulse force

The response of a non-damped electrical oscillator [10], [11], [12] subjected to a rectangular pulse force is obtained from the following equation

$$L\ddot{\mathrm{Q}}(\mathrm{t}) + \frac{1}{\mathrm{C}}\mathrm{Q}(\mathrm{t}) = \mathrm{V}(\mathrm{t})$$

Or

$$\ddot{\mathrm{Q}}(\mathrm{t}) + {\omega_0}^2\mathrm{Q}(\mathrm{t}) = \frac{\mathrm{V(t)}}{L} \ldots. (5)$$

$where\ \omega_0 = \sqrt{\frac{1}{\mathrm{LC}}}$, V(t) is a rectangular pulse potential. Also, [13] Q (0) = 0 & $\dot{\mathrm{Q}}(0) = 0$.

The RT of equation (5) provides

$q^2\overline{\mathrm{Q}}(\mathrm{q}) - q^4\mathrm{Q}(0) - q^3\dot{\mathrm{Q}}(0) + {\omega_0}^2\overline{\mathrm{Q}}(\mathrm{q}) = \frac{1}{L}q^3 \int_0^\infty e^{-qt}\,\mathrm{V(t)}\,dt$

Here $\overline{\mathrm{Q}}(\mathrm{q})$ is RT of Q(t).

Put $Q(0) = 0$ and $\dot{Q}(0) = 0$, we get

$$q^2\overline{Q}(q) + \omega_0{}^2\overline{Q}(q) = \frac{V_o}{L}q^3\int_0^{t_1} e^{-qt}\,dt + \frac{1}{L}q^3\int_{t_1}^{\infty} e^{-qt}\,(0)dt\}$$

$$q^2\overline{Q}(q) + \omega_0{}^2\overline{Q}(q) = \frac{V_o}{L}q^3\int_0^{t_1} e^{-qt}\,(1)\,dt$$

$$\Rightarrow q^2\overline{Q}(q) + \omega_0{}^2\overline{Q}(q) = \frac{V_o}{L}\{-q^2\,[e^{-qt_1} - 1]\}$$

$$\Rightarrow \overline{Q}(q) = \frac{V_o}{L}\{\frac{q^2}{(q^2 + \omega_0{}^2)} - \frac{q^2}{(q^2 + \omega_0{}^2)}e^{-qt_1}\}$$

$$\Rightarrow \overline{Q}(q) = \frac{V_o}{L}\{\frac{q^4}{q^2(q^2 + \omega_0{}^2)} - \frac{q^4}{q^2(q^2 + \omega_0{}^2)}e^{-qt_1}\}$$

$$\Rightarrow \overline{Q}(q) = \frac{V_o}{L}\{\frac{q^2}{(\omega_0{}^2)} - \frac{q^4}{(\omega_0{}^2)(q^2 + \omega_0{}^2)} - \frac{q^2}{(\omega_0{}^2)}e^{-qt_1} + \frac{q^4}{(\omega_0{}^2)(q^2 + \omega_0{}^2)}e^{-qt_1}\}$$

Taking inverse RT, we have

$$Q(t) = \frac{V_o}{L}\{\frac{1}{(\omega_0{}^2)} - \frac{\cos\omega t}{(\omega_0{}^2)} - \frac{1}{(\omega_0{}^2)}U(t - t_1) + \frac{\cos\omega(t - t_1)}{(\omega_0{}^2)}U(t - t_1)\}$$

$$\Rightarrow Q(t) = \frac{V_o}{L(\omega_0{}^2)}\{1 - \cos\omega_0 t - U(t - t_1) + \cos\omega_0(t - t_1)\,U(t - t_1)\}$$

$$\Rightarrow Q(t) = V_o C\,\{1 - \cos\omega_0 t - U(t - t_1) + \cos\omega_0(t - t_1)\,U(t - t_1)\} \ldots (6)$$

For $t < t_1$,

$$Q(t) = V_o C\{1 - \cos\omega_0 t\} \ldots\ldots (7a)$$

Taking $V_o = 300$V, C = 1 microfarad and ω_0=314rad/s, the graph of (7a) is shown in the figure below.

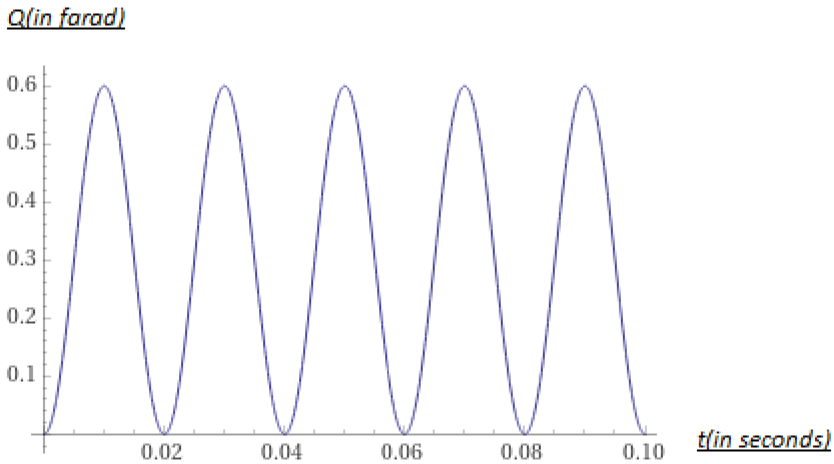

Figure 3: Respone of Non-damped Electrical Oscillator before removal of rectangular pulse force

For $t > t_1$,

$$Q(t) = V_o C\{-\cos\omega_0 t + \cos\omega_0(t - t_1)\}\ldots\ \ldots (7b)$$

Taking $V_o = 400$V, C = 0.5 microfarad, $t_1 = 0.1$s and ω_0=314rad/s, the graphs of (7b) is shown in the figure below.

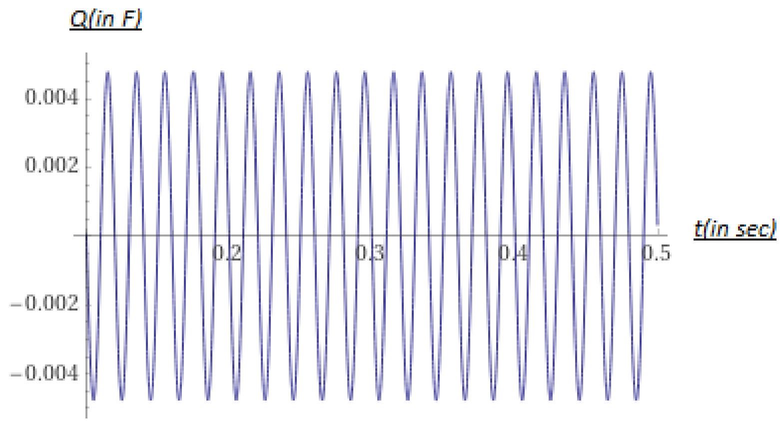

Figure 4: Respone of Non-damped Electrical Oscillator after removal of rectangular pulse force

3 RESULTS AND DISCUSSIONS

In the paper, the response of non-damped oscillators (i.e. electrical and mechanical oscillators) subjected to a rectangular pulse has been fortuitously fixed by the integral RT. The paper embellished the RT for fixing the the response of non-damped oscillators (i.e. electrical and mechanical oscillators) subjected to a rectangular pulse. In case of non-damped mechanical oscillators subjected to a rectangular pulse, it is clear that before the removal of rectangular pulse, the displacement of the oscillator increases and decreases periodically with constant amplitude as shown in the figure 1, but when we remove the rectangular pulse, the nature of displacement becomes oscillatory with constant amplitude as shown in the figure 2. Also, in case of non-damped electricall oscillator subjected to a rectangular pulse, it is clear that before the removal of rectangular pulse, the electric charge in the oscillator increases and decreases periodically with constant amplitude as shown in the figure 3, but when we remove the rectangular pulse, the nature of charge becomes oscillatory with constant amplitude as shown in the figure 4. The results fixed are the same as fixed with other methods [1], [2], [3], [14].

4 CONCLUSION

A novel method, that is, integral RT has been exploited for successfully finding the response of non-damped oscillators subjected to a rectangular pulse. This paper tenders an alternate style for finding the response of non-damped oscillators subjected to a rectangular pulse.

FUTURE SCOPE

In near future, RT is expected to be applied to fractional differential equations and in cryptography.

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