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An Efficient Logarithmic Ratio Type Estimator of Finite Population Mean under Simple Random Sampling

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ABSTRACT

The use of auxiliary information has become indispensable for improving the exact of estimators of population parameters like the mean and variance of the variable under study. A great variety of the techniques such as the ratio, product, and regression methods of estimation are commonly known in this esteem. In this paper, we propose an efficient logarithmic ratio type estimator for finite population mean estimation under simple random sampling. The expression for the bias and mean squared error (MSE) of the proposed estimator is obtained up to the first order of approximation. The conditions under which the proposed estimator is more efficient than the existing ones are established. An empirical study using three data sets is also conducted to validate the theoretical findings and the results revealed that the suggested estimator is better than the existing estimators considered in the study.

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1. INTRODUCTION

Sampling survey is a field of statistics that deals with methods of sampling and estimation from sample, it is essential to use sampling to reduce cost and time. At the design stage or the estimation stage or both stages, auxiliary information is used to increase the precision of the estimators of population parameters. In this context, ratio, exponential ratio, product, exponential product, and regression methods of estimation are used for parameter estimation. In statistics, the coefficient of variation is used to make a comparison between two or more things with different units or dimensions, for example, the weight and height of individuals, are in kilograms (kg) and centimeters (cm). Auxiliary variables are often used in sample survey to obtain improved precision of the estimate of the population parameters such as the population mean, population variance, and population coefficient of variation of the study variable. This information may be used at both the design and estimation stages. Ratio, regression, and product methods of estimation are used in this context. Researchers have constructed estimators for the estimation of parameters by modifying existing estimators using the known function of an auxiliary variable, authors like Cochran[1], Srivastava [2], Sisodia and Dwivedi [3], Bahl and Tuteja [4], Singh and Tailor [5], Singh et al [6], Kadilar and Cingi [7], Singh and Tailor [8], Singh et al [9], Singh and Solanki [10], Brar and Kaur [11], Brar et al [12], Muili et al [13], Audu and Singh[14], Yunusa et al.[15], Audu et al.[16], Adejumobi et al.[19], Singh [20, 21], Khosnevisan et al [22], Perri [23], Upadhyaya et al [24], Adejumobi and Yunusa [25], Singh and Kumar [26], Adebola and Adegoke [27].

In this current study, we proposed an efficient logarithmic ratio type estimator for finite population mean estimation under simple random sampling. The proposed estimators are expected to perform better than the existing related estimators considered in the literature and to produce estimates closer to the true population mean.

Consider a finite population $U=\left(U_1,U_2,......,U_N\right)$. We draw a sample size n from the population using a simple random sampling without replacement (SRSWOR) scheme. Let y and x respectively be the study and the auxiliary variables and y_i and x_i, be the observations on the i[th] unit. Let $\bar{y}=n^{-1}\sum_{i=1}^{n}y_i$ and $\bar{x}=n^{-1}\sum_{i=1}^{n}x_i$ be the sample means and $\bar{Y}=N^{-1}\sum_{i=1}^{N}y_i$ and $\bar{X}=N^{-1}\sum_{i=1}^{N}x_i$, be the corresponding population means of the study and auxiliary variables respectively. Let $s_y^2=\left(n-1\right)^{-1}\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2$ and $s_x^2=\left(n-1\right)^{-1}\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2$ be the sample variances and $S_y^2=\left(N-1\right)^{-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2$ and $S_x^2=\left(N-1\right)^{-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2$, be the corresponding population variances. Let ρ be the correlation coefficient between y and x. Finally let $C_y=S_y\bar{Y}^{-1}$ and $C_x=S_x\bar{X}^{-1}$ respectively be the coefficients of variation for y and x.

To obtain the bias and mean squared error (MSE) for the proposed estimators and existing ones considered here, we defined the following sampling error terms.

Let $e_0=\bar{Y}^{-1}\left(\bar{y}-\bar{Y}\right)$ and $e_0=\bar{Y}^{-1}\left(\bar{y}-\bar{Y}\right)$ such that $E\left(e_i\right)=0$ for $\left(i=0,1\right)$, $E\left(e_0^2\right)=\lambda C_y^2$, $E\left(e_0^2\right)=\lambda C_y^2$ and $E\left(e_0^2\right)=\lambda C_y^2$, where $\lambda=\left(n^{-1}-N^{-1}\right)$.

In this paper, we propose an efficient logarithmic ratio type estimator for the estimation of finite population mean that is more efficient than the existing estimators.

1.1 Some Existing Estimators in Literature

In this section, we consider some existing estimators of finite population mean.

The usual sample mean is defined as:

$$\bar{y}=n^{-1}\sum_{i=1}^{n}y_i \tag{1}$$

The variance of the estimator $\bar{y}$ is given by

$$Var\left(\bar{y}\right)=\lambda\bar{Y}^2C_y^2 \tag{2}$$

Cochran [1] suggested a ratio estimator and it is defined as:

$$T_R=\left(\frac{\bar{y}}{\bar{x}}\right)\bar{X} \tag{3}$$

The MSE of T_R, to the first order of approximation is given by

$$MSE(T_R)=\lambda\bar{Y}^2(C_y^2+C_x^2-2\rho C_yC_x) \tag{4}$$

Bahl and Tuteja[4] suggested ratio and product type estimators of $\bar{Y}$, given respectively as:

$$T_{BTR}=\bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \tag{5}$$

The MSE of T_{BTR} to first order of approximation is given by:

$$MSE(T_{BTR})=\lambda\bar{Y}^2\left[C_y^2+\frac{C_x^2}{4}-\rho C_yC_x\right] \tag{6}$$

Brar et al. [12] proposed new functional forms of ratio estimator of the population mean as:

$$\bar{y}_{LR}=\bar{y}\frac{\bar{X}}{\bar{x}-\bar{X}}Ln\left(\frac{\bar{x}}{\bar{X}}\right) \tag{7}$$

The mean square error (MSE) of the estimator is given by:

$$MSE(\bar{y}_{LR}) = \lambda \bar{Y}^2 \left[C_y^2 + \frac{C_x^2}{4} - \rho C_y C_x \right] \tag{8}$$

Yunusa et al. [28] implies a logarithmic-product-cum-ratio type estimator for estimating coefficient of variation as;

$$T_{am} = \hat{C}_y \left(\frac{Ln(\bar{x})}{Ln(\bar{X})} \right) \left(\frac{Ln(S_x^2)}{Ln(s_x^2)} \right) \tag{9}$$

The mean square error (MSE) of the suggested estimator is known as;

$$MSE(T_{am}) = C_y{}^2 \gamma \left(\begin{array}{l} \frac{(\lambda_{40} - 1)}{4} + C_y{}^2 + \theta_1^2 C_x^2 + \theta_2^2 (\lambda_{04} - 1) - C_y \lambda_{30} + \theta_1 C_x \lambda_{21} - \theta_2 (\lambda_{22} - 1) \\ -2\theta_1 \rho C_y C_x + 2\theta_2 C_y \lambda_{12} - 2\theta_1 \theta_2 C_x \lambda_{03} \end{array} \right) \tag{10}$$

2. RESEARCH METHOD

2.1 Proposed Estimator

Motivated by Cochran [1] and Brar et al. [12], we proposed new logarithmic ratio type estimator for estimating finite population mean as:

$$\hat{\theta} = \frac{\left[\bar{y} - Ln\left(\frac{\bar{x}}{\bar{X}} \right) \right]}{\bar{x}} \bar{X} \tag{11}$$

Expressing estimator $\hat{\theta}$ in terms of error terms, we have:

$$\hat{\theta} = \frac{\left[\bar{Y}(1+e_0) - Ln\left(\frac{\bar{X}(1+e_1)}{\bar{X}} \right) \right]}{\bar{X}(1+e_1)} \bar{X} \tag{12}$$

$$\hat{\theta} = \left[\bar{Y}(1+e_0) - Ln(1+e_1) \right] (1+e_1)^{-1} \tag{13}$$

$$\hat{\theta} = \left[\bar{Y}(1+e_0) - \left(e_1 - \frac{e_1^2}{2} \right) \right] \left(1 - e_1 + e_1^2 \right) \tag{14}$$

Expanding the right-hand side of equation (14), we have,

$$\hat{\theta} = \bar{Y}\left(1 - e_1 + e_1^2 + e_0 - e_0 e_1 \right) - \left(e_1 - \frac{3e_1^2}{2} \right) \tag{15}$$

Subtract $\bar{Y}$ from both sides of equation (15), we have,

$$\hat{\theta} - \bar{Y} = \bar{Y}\left(e_0 - e_1 + e_1^2 - e_0 e_1 \right) - \left(e_1 - \frac{3e_1^2}{2} \right) \tag{16}$$

Taking expectation of both sides of equation (16) to obtain the bias of $\hat{\theta}$ to first order of approximation, we have,

$$Bias(\hat{\theta}) = \lambda \left[\bar{Y}\left(C_x^2 - \rho C_y C_x \right) + \frac{3C_x^2}{2} \right] \tag{17}$$

By squaring and taking expectation of equation (16) to obtain the MSE of estimator $\hat{\theta}_1$ as:

$$MSE(\hat{\theta}) = \lambda \left[\bar{Y}^2 \left(C_y^2 + C_x^2 - 2\rho C_y C_x \right) + \left(1 + 2\bar{Y} \right) C_x^2 - 2\bar{Y} \rho C_y C_x \right] \tag{18}$$

2.2 Efficiency Comparisons

In this section, conditions for the efficiency of the new estimators over some existing estimators were established.

Estimator $\hat{\theta}$ is more efficient than the estimator $\bar{y}$ if:

$$Var(\bar{y}) - MSE(\hat{\theta}) > 0 \tag{19}$$

$$\rho > \frac{C_x(\bar{Y}+1)}{2\bar{Y}C_y} \tag{20}$$

Estimator $\hat{\theta}$ is more efficient than the estimator t_R if:

$$MSE(t_R) - MSE(\hat{\theta}) > 0 \tag{21}$$

$$\rho > \frac{C_x(2\bar{Y}+1)}{2\bar{Y}C_y} \tag{22}$$

Estimator $\hat{\theta}$ is more efficient than the estimator t_{BTR} and $\bar{y}_{LR}$ if:

$$MSE(t_{BTR}) = MSE(\bar{y}_{LR}) - MSE(\hat{\theta}) > 0 \tag{23}$$

$$\rho > \frac{C_x(3\bar{Y}+2)}{4\bar{Y}C_y} \tag{24}$$

3. RESULTS AND DISCUSSIONS

In this section, numerical analyses to elucidate the performance of the estimators are illustrated. The population 1 is taken from page 399 of Murthy [18], populations 2 and 3 are taken from page 177 of Singh and Chaudhary [17] and from Kadilar and Cingi [7].

Population 1: Murthy [18]

$$N = 34, n = 15, \bar{X} = 208.88, \bar{Y} = 199.44, C_x = 0.72, C_y = 0.75, \rho = 0.98$$

Population 2: Singh and Chaudhary [17]

$$N = 204, n = 50, \bar{X} = 26441, \bar{Y} = 966, C_x = 1.7171, C_y = 2.4739, \rho = 0.71$$

Population 2: Kadilar and Cingi [7]

$$N = 106, n = 40, \bar{X} = 27421.70, \bar{Y} = 2212.59, C_x = 2.10, C_y = 5.22, \rho = 0.860$$

Table 1: MSE and PRE of Proposed and Existing Estimators

Estimators	Population 1		Population 2		Population 3	
	MSE	PRE	MSE	PRE	MSE	PRE
$\bar{y}$	833.5477	100.00	86226.17	100.00	12081150	100.00
t_R	33.3419	2500	42781.39	201.5506	5676816	212.816
t_{BTR}	241.3954	345.304	42779.47	103.366	8390166	143.992
$\bar{y}_{LR}$	241.3954	345.304	54118.79	103.366	8390166	143.992
$\hat{\theta}$	**33.20073**	**2510.631**	**1375526**	**106.641**	**5674806**	**212.891**

Table 1 shows MSEs and PREs of the proposed estimator and existing estimators. The results showed that the prpostedd estimator has minimum MSE and higher PRE among other estimators for the three populations considered in the study. These results imply that the proposed estimator is more efficient than the existing ones considered in this study.

4. CONCLUSION

In this study, an efficient logarithmic ratio type estimator proposed for the estimation of finite population mean has been suggested. The empirical study was conducted to reveal the superiority of the logarithmic ratio type estimator over its existing counterpart and the results revealed that the new proposed estimator outperformed estimators considered in the literature. This show that the proposed estimator can produce better estimate than conventional and existing related estimators. Hence, the new estimator is recommended for use in real life situations.

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