International Journal of Engineering and Applied Physics (IJEAP)
Vol. 3, No. 1, January 2023, pp. 595~599
ISSN: 2737-8071



# Approximation of the buckling load in functionally graded beams with two types of porous distribution

**Lan Hoang That Ton**
Dept. of Civil Engineering at HCMC University of Architecture, HCMC, Vietnam.

**Article Info**

***Article history:***

Received Aug 18, 2022
Revised Sept 12, 2022
Accepted Sept 24, 2022

***Keywords:***

Buckling load
Functionally graded porous beam
FEM
Porous distribution
Finite element method

**ABSTRACT**

For the purpose of studying the buckling behavior of functionally graded porous beams with one kind of porous distribution, a short article is given. Although the research topic is nothing special, based on the simple beam model related to the finite element method and MATLAB code, we can calculate the approximation of the buckling load of the beam under the influence of porosity. An example of buckling load estimates for this structure is given, which shows that our method is applicable to a certain extent.



***Corresponding Author:***

Lan Hoang That Ton,
Department of Civil Engineering,
HCMC University of Architecture,
HCMC, Vietnam.
Email: tltechonlinesom@gmail.com

## 1. INTRODUCTION

Today, functionally graded materials have been widely used with many outstanding advantages. However, the fabrication process often leads to the appearance of voids inside the material. This leads to unpredictability in mechanical behavior and forces the designer to have insight into its mechanical properties [1-4]. Besides, numerical methods, typically the finite element method, have demonstrated an outstanding ability at calculating the behavior of the structure. Noori et al. [5] introduced an efficient numerical procedure to the solution of the dynamic response of functionally graded porous beams. By using the canonically conjugate momentums and their derivatives, the governing canonical equations of motion of these beams were derived. These equations were then transformed into the Laplace space and solved numerically with the aid of the complementary function's method. Another study was to investigate the buckling characteristics of a sandwich beam consisting of a porous ceramic core (Alumina), two bottom and upper layers which were gradually changed from ceramic (Alumina) to metal (Aluminum) through the thickness direction [6]. Shear deformation effects were taken into account based on third-order shear deformation theory and a two-variable refined shear deformation theory. In the literature [7], vibration analysis of functionally graded porous beams was carried out using the third-order shear deformation theory. The beams had uniform and non-uniform porosity distributions across their thickness and both ends were supported by rotational and translational springs. The Chebyshev collocation method was also applied to solve the governing equations derived from Hamilton's principle, which is used in order to obtain the accurate natural frequencies for the vibration problem of beams with various general and elastic boundary conditions. The paper of Chen *et al.* [8] presented the elastic buckling and static bending analysis of shear deformable functionally graded (FG) porous beams. The elasticity moduli and mass density of porous composites were assumed to be graded in the thickness direction. The Ritz method was employed to obtain the critical buckling loads, where the trial functions take the form of simple algebraic polynomials. Another efficient numerical procedure was introduced to the solution of the dynamic response of functionally graded porous beams by Noori et al. [9]. Within the framework of the first-order shear deformation theory, the influence of shear strain was included in the formulations. The impact of damping was also considered. By using the canonically conjugate momentums and their derivatives, the governing canonical equations of motion of FGP beams were derived for the first time. These equations were then transformed into the Laplace space and solved numerically with the aid of the Complementary Functions Method, etc. The article is built with the aim of evaluating the influence of porosity on the buckling behavior of functionally graded porous beams. Following the introduction are the sections of formulas related to materials, finite element results and conclusions.

## 2. FORMULATIONS

A functionally graded porous beam with length *L*, thickness *h* and breadth *b* is studied, as in Figure 1, for two types of porosity.

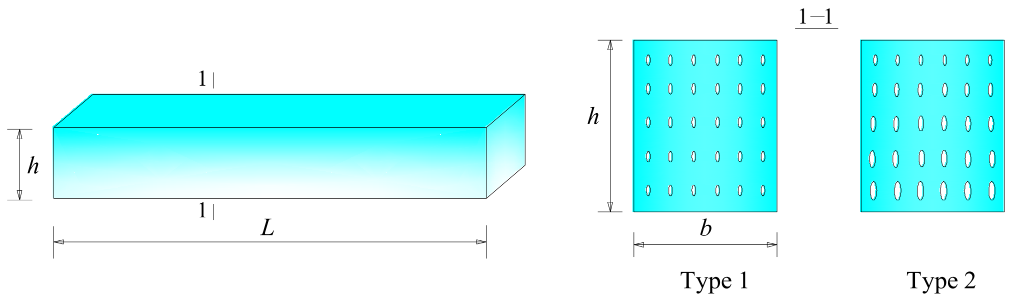


Figure 1. A functionally graded porous beam with two types of porosity.

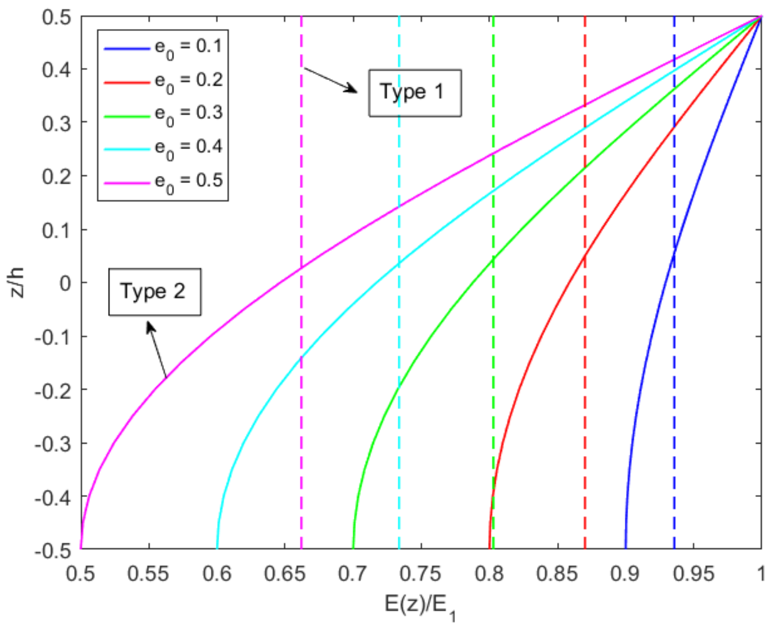


Figure 2. Normalized material property $E(z)/E_1$

The material properties are assumed to vary in thickness direction according to [8, 9]

Type 1
$$E(z)=E_1\left[1-e_0\Psi\right],\quad G(z)=G_1\left[1-e_0\Psi\right], \tag{1}$$

with
$$\Psi=\frac{1}{e_0}-\frac{1}{e_0}\left(\frac{2}{\pi}\sqrt{1-e_0}-\frac{2}{\pi}+1\right)^2$$

Type 2
$$E(z)=E_1\left[1-e_0\cos\left(\frac{\pi z}{2h}+\frac{\pi}{4}\right)\right],\quad G(z)=G_1\left[1-e_0\cos\left(\frac{\pi z}{2h}+\frac{\pi}{4}\right)\right], \tag{2}$$

with
$$e_0=1-\frac{E_2}{E_1}=1-\frac{G_2}{G_1}$$

The ratio $E(z)/E_1$ is shown in Figure 2 to make clear the influences of porosity on the value of Young's modulus at the top surface, $E_1$. Note that $0 < e_0 < 1$ in Eqs. (1-2)

## 3. RESULTS AND DISCUSSIONS

Firstly, an isotropic beam with $L = 5$ m, $b = h = 0.25$ m and $E = 2\times10^{11}$ Pa is verified. By changing four kinds of boundary condition, such as C – F (*clamped – free*), C – C (*clamped – clamped*), C – S (*clamped – simply supported*) and S – S (*simply supported – simply supported*), the buckling load is calculated based on a finite element method, Matlab code and shown in Table 1. These values are compared with the other solutions related to theory [10]. The errors between the results are not significant respectively. Besides, the first four buckling mode shapes are also plotted in Figure 3.

Table 1. Verification of the buckling load (kN)

| *BCs* | *FEM* | *Theory* [10] |
|---|---|---|
| C-F | 6425.52 | 6425.5 |
| C-C | 102808.38 | 102808.4 |
| C-S | 52580.02 | 52580 |
| S-S | 25702.09 | 25702.1 |

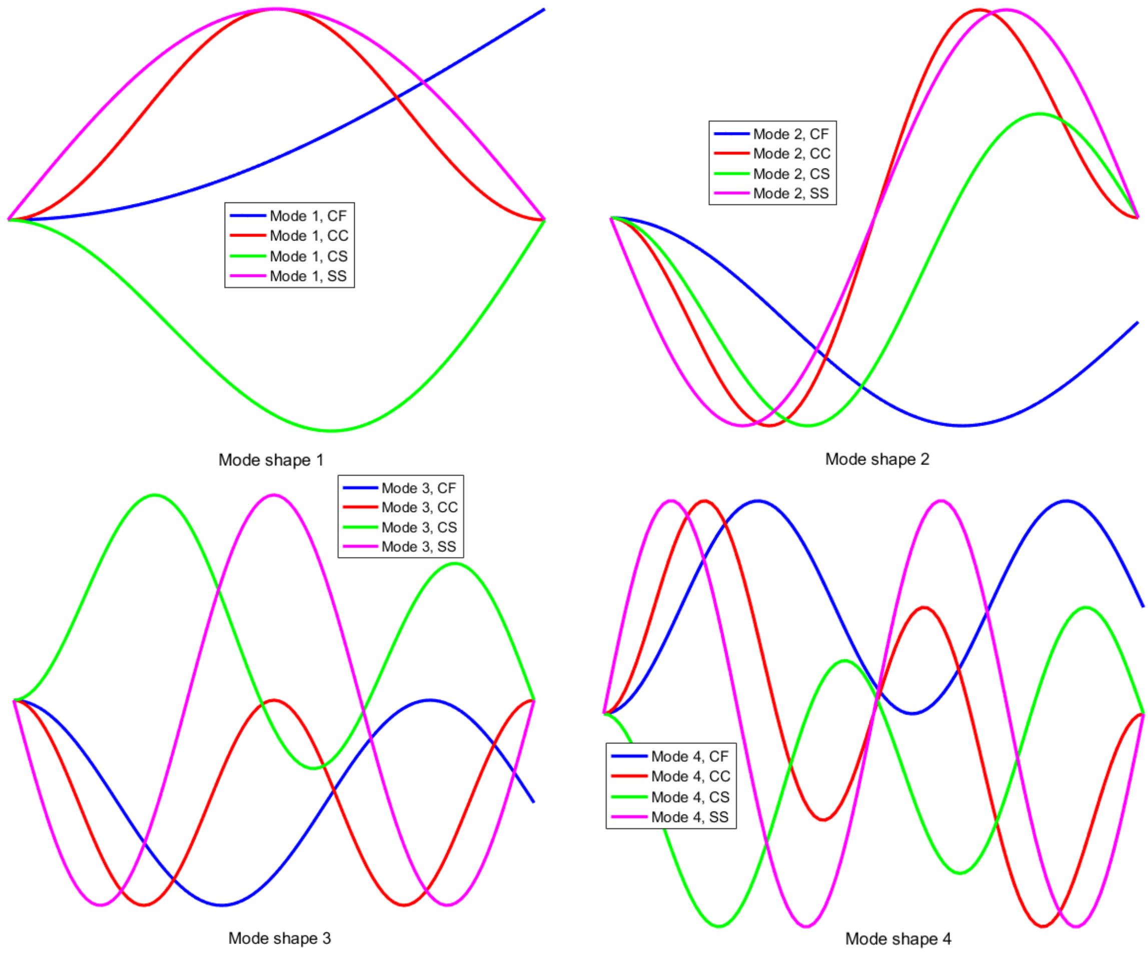


Figure 3. First four buckling mode shapes

The functionally graded porous beam is now considered in this article with $E_1 = 2\times10^{11}$ Pa and two types of porous distribution as depicted in Figure 1. By changing factor $e_0$ and boundary condition, the buckling loads for all cases are estimated and displayed in Table 2 and Figure 4 respectively. Their values for type 2 are slightly larger than for type 1, and, of course, when the coefficient $e_0$ is increased, the buckling loads decrease. Regarding the difference in porosity distribution between types 1 and 2, the results obtained are also different. The above influences should be taken into account when designing structures made of this material.

Table 2. The buckling load (kN) by changing $e_0$

| | | | Type 1 | | |
|---|---|---|---|---|---|
| $e_0$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| C-F | 6012.62 | 5590.90 | 5158.74 | 4713.81 | 4252.75 |
| C-C | 96201.92 | 89454.41 | 82539.88 | 75421.00 | 68043.99 |
| C-S | 49201.23 | 45750.30 | 42213.96 | 38573.10 | 34800.23 |
| S-S | 24050.48 | 22363.60 | 20634.97 | 18855.25 | 17010.99 |
| | | | Type 2 | | |
| $e_0$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| C-F | 6052.28 | 5678.97 | 5305.66 | 4932.34 | 4559.03 |
| C-C | 96836.58 | 90863.54 | 84890.50 | 78917.46 | 72944.42 |
| C-S | 49525.82 | 46470.97 | 43416.15 | 40361.32 | 37306.49 |
| S-S | 24209.14 | 22715.88 | 21222.62 | 19729.36 | 18236.11 |

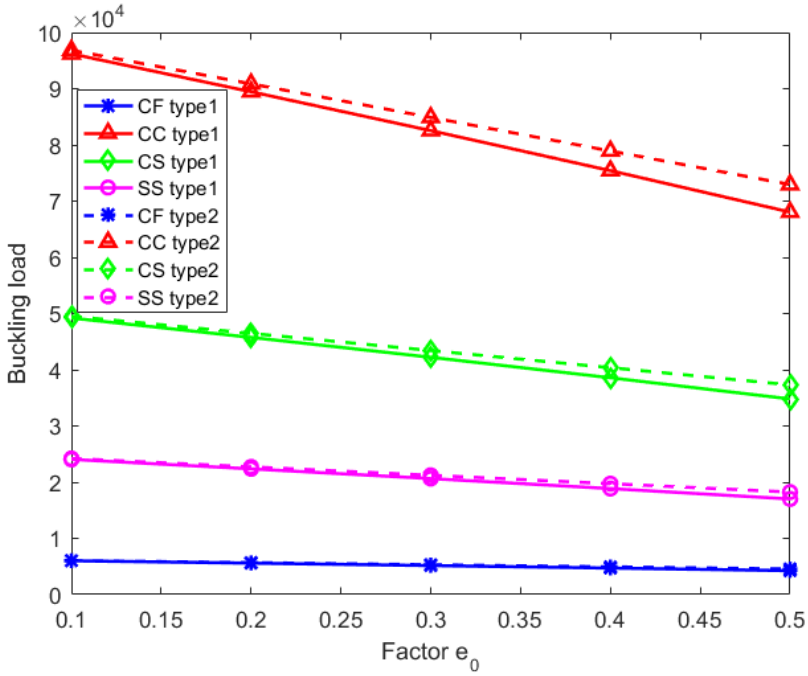

Figure 4. Comparison of the buckling load with two types of porosity

## 4. CONCLUSION

This short article shows the buckling load of functionally graded porous beams with varying boundary conditions. Based on MATLAB software and the finite element method with a simple beam model, we can calculate the approximation of these buckling loads for C-F, C-C, C-S, S-S beams under the influence of porosity. An example of estimating the buckling load for this structure is given, which shows that our method can apply with acceptable results.

## REFERENCES

[1] N. Zhang, T. Khan, H. Guo, S. Shi, W. Zhong, and W. Zhang, "Functionally Graded Materials: An Overview of Stability, Buckling, and Free Vibration Analysis," *Advances in Materials Science and Engineering,* vol. 2019, p. 1354150, 2019.

[2] B. Kieback, A. Neubrand, and H. Riedel, "Processing techniques for functionally graded materials," *Materials Science and Engineering: A,* vol. 362, pp. 81-106, 2003.

[3] M. Naebe and K. Shirvanimoghaddam, "Functionally graded materials: A review of fabrication and properties," *Applied Materials Today,* vol. 5, pp. 223-245, 2016.

[4] H. Wu, J. Yang, and S. Kitipornchai, "Mechanical Analysis of Functionally Graded Porous Structures: A Review," *International Journal of Structural Stability and Dynamics,* vol. 20, p. 2041015, 2020.

[5] A. R. Noori, T. A. Aslan, and B. Temel, "Dynamic Analysis of Functionally Graded Porous Beams Using Complementary Functions Method in the Laplace Domain," *Composite Structures,* vol. 256, p. 113094, 2021.

[6] M. Derikvand, F. Farhatnia, and D. H. Hodges, "Functionally graded thick sandwich beams with porous core: Buckling analysis via differential transform method," *Mechanics Based Design of Structures and Machines,* pp. 1-28, 2021.

[7] N. Wattanasakulpong, A. Chaikittiratana, and S. Pornpeerakeat, "Chebyshev collocation approach for vibration analysis of functionally graded porous beams based on third-order shear deformation theory," *Acta Mechanica Sinica,* vol. 34, pp. 1124-1135, 2018.

[8] D. Chen, J. Yang, S. Kitipornchai, "Elastic Buckling and Static Bending of Shear Deformable Functionally Graded Porous Beam," *Composite Structures,* vol. 133, pp. 54-61, 2015.

[9] A. R. Noori, T. A. Aslan, B. Temel, "Dynamic Analysis of Functionally Graded Porous Beams Using Complementary Functions Method in the Laplace Domain", *Composite Structures,* vol. 256, p. 113094, 2021.

[10] https://web.mit.edu/.